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Optimization of Heat Transfer from Radiating Fins of Rectangular, Trapezoidal and Parabolic Geometry

by

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May, 1999

Optimization of Heat Transfer from Radiating Fins of Rectangular, Trapezoidal and Parabolic Geometry

A Thesis Submitted

in Partial Fulfilment of the Requirements

for the Degree of

Master of Technology

by

Nrependra Kumar

to the

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

May, 1999

CERTIFICATE

It is certified that the work contained in the thesis entitled **Optimization of Heat Transfer from Radiating fins of Rectangular, Trapezoidal and Parabolic Geometry** by **Nrependra Kumar (Roll no. 9720514)**, has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

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Acknowledgements

I would like to expresss my profound gratitude to my thesis supervisor Dr.P.S.Ghoshdastidar for his unending support and encouragement given to me throughout my thesis work.I feel he is the most disciplined and punctual person I have ever met and I am sure the discipline I learned from him will guide me througout my life.

I would like to express my sincere thanks to my best friend Alam,who has helped me to debug most of my programs.I would also like to thank Bhargava for teaching me lyx in a lucid manner.Last but not the least I would like to thank all my friends,especially Chandu,Amit,Sandeep Jain etc who has made my stay in IIT pleasant .

Nrependra kumar.

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ABSTRACT

In outer space , the only means of rejecting heat is that of thermal radiation . For this purpose heat rejection space radiators have been developed. Internally generated heat within the space craft is conveyed to the radiator by a circulating coolant (e.g.,a water-glycol mixture). The coolant circulates in tubes (usually in a parallel circuit) which are joined by finned surfaces. Heat from the coolant conducts into the fin and is dissipated by the radiation to the surroundings . The finned surface may be subjected to an external irradiation from a source such as the sun .

In this work , a finite- difference solution is obtained for 1D radiating fins of rectangular, parabolic and trapezoidal profile,the material being C-steel, aluminium and copper. It is also assumed that the conductivity is a function of temperature . The results for the limiting case of constant conductivity rectangular fin have been checked with the earlier solution of NASA and found very satisfactory. Based on simulation results, that is , the fin temperature distribution and the heat transfer , an optimization study has been carried out with regard to the fin geometry and dimensions as well

as materials in order to maximize heat transfer from the fin. An exhaustive search method has been used. It is found that aluminium is the most suitable material. Also, trapezoidal and parabolic fins are recommended for efficient heat transfer.

Nomenclature

a	profile of parabolic fin is $y = a + bx + cx^2$
b	profile of parabolic fin is $y = a + bx + cx^2$
c	profile of parabolic fin is $y = a + bx + cx^2$
C_1	$= \frac{W}{\sigma\epsilon L^2 t_0^3}$
C_2	$= \frac{1.0}{\sigma\epsilon L^2 t_0^3}$
G	external irradiation
k	thermal conductivity of material
k_i	is the thermal conductivity of material at i^{th} grid point
T	non dimensional temperature
T_i	non dimensional temperature at i^{th} grid point
T_{oi}	non dimensional temperature at i^{th} grid point in previous iteration

t_0	base temperature of the fin
t_s^4	$=\frac{\alpha G}{\sigma \epsilon}$
V	volume of the fin
W	width of rectangular fin
W_1	width of trapezoidal fin at base
W_2	width of trapezoidal fin at tip
X	non dimensional distance toward the fin
x	distance

Greek letters

α	absorptivity
ϵ	emissivity
σ	Stefan - Boltzmann's constant $5.67 \times 10^{-8} W/m^2.K^4$
ρ	density of material

Chapter 1

INTRODUCTION

1.1 GENERAL REMARKS

One of the most important problems of modern space technology is that of thermal control-the maintenance of a desired temperature level with in space craft (manned or unmanned) or satellite . Such spacecraft subjected to heat input of both internal and external sources . Internal sources consist of electrically generated heat , while the external sources consist of thermal irradiation from the sun , planetary bodies ,etc . If a net gain of heat is experienced, some means of heat rejection must be provided in order that thermal control be maintained.

In outer space , the only means of rejecting heat is that of thermal radiation . For this purpose heat rejection space radiators have been developed. A typical space radiator configuration is shown in Fig 1.1.

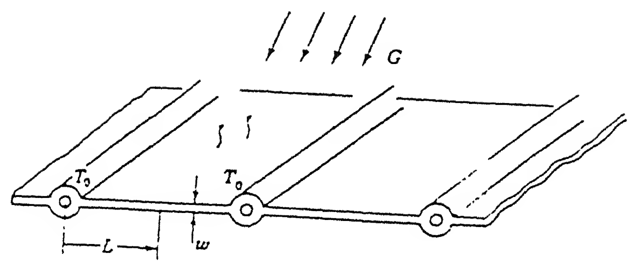

Figure 1.1: Heat rejection space radiator

Internally generated heat with in space craft is conveyed to the radiator by a circulating coolant (e.g., a water glycol mixture) . The coolant circulates in tubes (usually connected in parallel circuit) which are joined by the finned surfaces . Heat from the coolant conducts into the fin and dissipated by radiation to the surroundings. The finned surface may subjected to an external irradiation from a source such as sun .The basic question to be answered is: Given a radiating fin of known geometry, maintained at a base (i.e.tube temperature) and subjected to external radiation, what is the rate at which heat is dissipated by the fin to the environment?

1.2 LITERATURE REVIEW

Optimum design of radiating fins has been the subject of research by a number of investigators. Wilkins' (1960) analysis of a single fin freely radiating to space shows that the profile of the fin of least material follows a power

law,i.e.$y \sim x^{\alpha}$, where y is the fin thickness at the length x from the tip of the fin and $\alpha = 3.5$. Sparrow et al. (1961) presented the optimum design of radiatively interacting longitudinal fins without considering fin -to -base mutual irradiation. Later Sparrow and Eckert (1962) emphasized the importance of mutual radiation interaction between the fin and its base surface. The work by Kerlekar and Chao (1963) also neglected fin-to-base interaction while analyzing the optimum design of trapezoidal fin. Schnurr et al. (1976) employed a nonlinear optimization technique to determine the minimum weight design for straight and circular fins of rectangular and triangular profiles, protruding from a cylinder, considering fin-to-fin and fin-to-base radiation interactions . Chung and Zhang (1991a) determined the optimum shape and minimum mass of a thin fin accounting for fin - to - base interaction based on a variational calculus approach . Chung and Zhang (1991b) later extended their analysis to minimize the weight of a radiating fin array projecting from a cylindrical surface considering both fin-to-fin and fin to base interactions . The results are presented in form of graphs and no functional form is given for the optimum profile of the fin. Krishnaprakas (1996) presented the optimum design of a rectangular plate fin array extending from a plane wall employing a nonlinear optimization method . Krishnaprakas (1997) presented the optimum length of a radiating longitudinal fin array extending from a cylindrical surface. Chung et al. (1996) analyzed and optimized a new designed for a four-fin radiating array in a fuzzy environment. Here, the objective function is not precisely defined , rather the two conflicting goals of weight reduction

and horizontal height minimization for space saving are fuzzily defined as fairly light and reasonably short.

1.3 OBJECTIVES

Radiating fins play an important role in thermal control of space vehicles and satellites. Since the weight and material are the major design considerations in heat exchangers , it is highly desirable to obtain an optimum design with respect to fin mass. The objectives of this study are to numerically simulate heat transfer from radiating fins of rectangular, parabolic and trapezoidal profiles and find the dimensions of the fins of different materials for maximum heat transfer.

Chapter 2

PROBLEM FORMULATION AND METHOD OF SOLUTION

2.1 PROBLEM STATEMENT

If the fluid in adjacent radiator tubes is at same temperature, the fin may represented by the models in the Fig2.1,Fig2.2 and Fig2.3. The base temperature of the fin is t_0 . The other end of the fin has the condition $\frac{dt}{dx} = 0.0$. The fin material has the thermal conductivity k which may be function of temperature, and expose surface has total emissivity ϵ . The surface is exposed to an external irradiation G (perhaps solar) to which it exhibits a total absorptivity α, since such radiators are often made an integral part of space craft skin, only one of the surface of fin is considered to be radiatively active .

2.2 GOVERNING DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

2.2.1 FOR RECTANGULAR FIN

If the conduction in the is fin taken to be one-dimensional, then the heat balance taken on an element δx in length yields the following relation (for a unit depth) of the rectangular fin of length L and uniform thickness W (Fig.2.1)

$$-kW\frac{dt}{dx}\mid_x = -kW\frac{dt}{dx}\mid_{x+\delta x} + \epsilon\sigma t^4 \delta x - \alpha G \delta x \tag{2.1}$$

'k ' may be also function of temperature. The actual functions of various fin materials used in this study are shown in Appendix 'B'.

No convective loss at the fin surface is included. For $\delta x \rightarrow 0.0$, the following differential equation must be satisfied for each point.

$$\frac{d}{dx}(kW\frac{dt}{dx}) = \epsilon\sigma(t^4 - \frac{\alpha G}{\epsilon\sigma}) \tag{2.2}$$

The non-homogeneous term is recognized as the equilibrium temperature, or equivalent sink temperature, defined in equation. This temperature is the equilibrium temperature the surface would achieve if isolated in irradiation G. Thus with

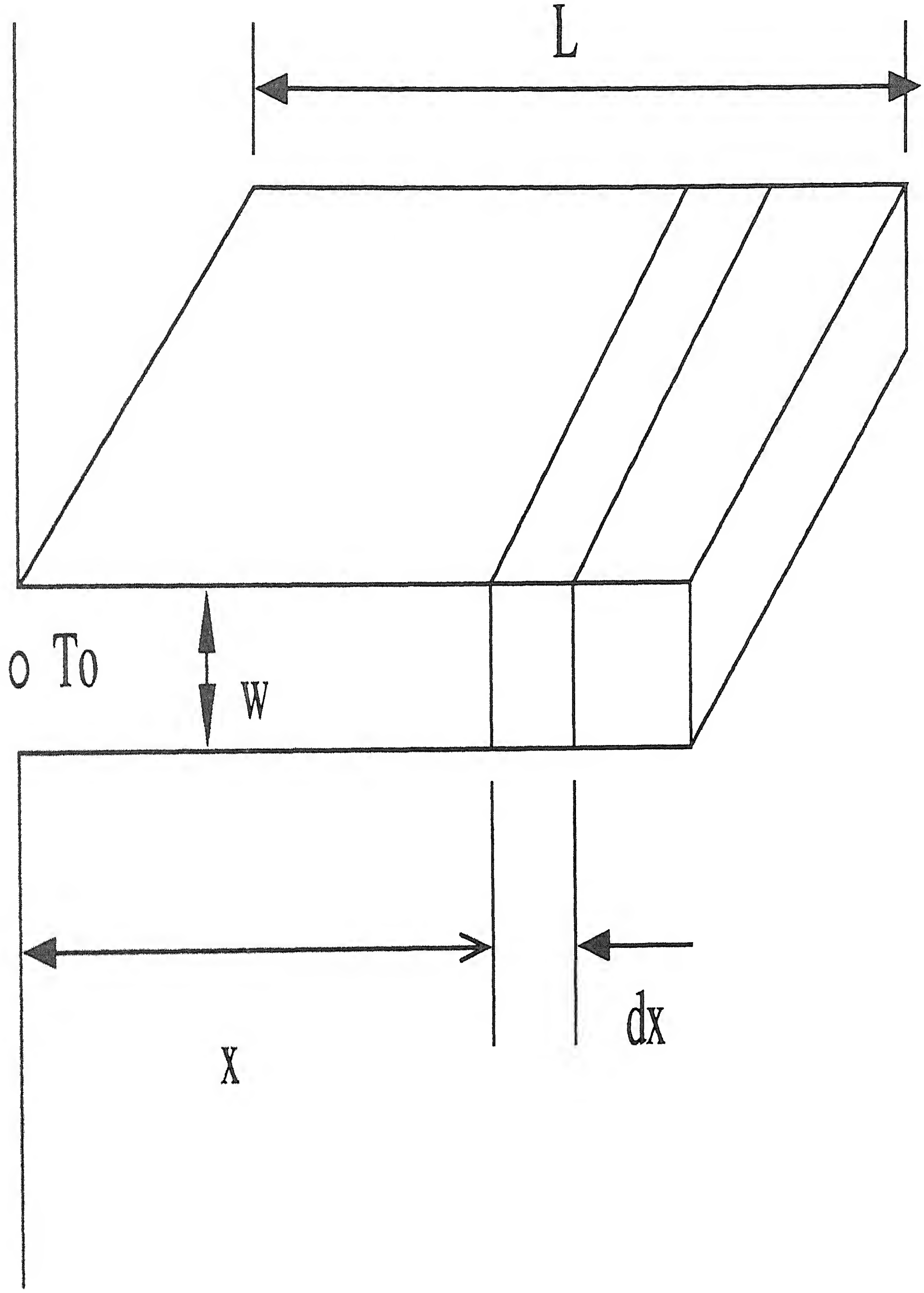

Figure 2.1: RECTANGULAR FIN

$$t_s^4 = \frac{\alpha G}{\epsilon\sigma} \tag{2.3}$$

The differential equation for the temperature distribution in the fin is

$$\frac{d}{dx}(kW\frac{dt}{dx}) - \epsilon\sigma(t^4 - t_s^4) = 0.0 \tag{2.4}$$

Boundary conditions are

At $x = 0.0$

$$t = t_0 \tag{2.5}$$

At $x = L$

$$\frac{dt}{dx} = 0.0 \tag{2.6}$$

The solution of above equation is best discussed in terms of the following dimensionless variables.

$$X = \frac{x}{L}$$

$$T = \frac{t}{t_0}$$

$$T_s = \frac{t_s}{t_0}$$

Introduction of these definitions in eq. (2.4) yields

$$(\frac{1.0}{\epsilon\sigma L^2 t_0^3})\frac{d}{dT}(kW\frac{dT}{dX}) - (T^4 - T_s^4) = 0.0 \tag{2.7}$$

The dimensionless boundary conditions to be satisfied are

At $X = 0.0$

$$T = 1.0 \tag{2.8}$$

At $X = 1.0$

$$\frac{dT}{dX} = 0.0 \tag{2.9}$$

2.2.2 FOR TRAPEZOIDAL FIN

If the conduction in the fin is taken to be one-dimensional, then the heat balance taken on an element δx in length yields the following relation (for a unit depth) for the trapezoidal fin of length L .The fin width at $x = 0.0$ is W_1 and $x = L$ is W_2 . See Fig. 2.2.

'k ' may be also function of temperature. The actual functions of various fin materials used in this study are shown in Appendix 'B'.

$$-k(W_1-(W_1-W_2)\frac{x}{L})\frac{dt}{dx}\mid_x = -k(W_1-(W_1-W_2)\frac{x}{L})\frac{dt}{dx}\mid_{x+\delta x} + \epsilon\sigma t^4\delta x - \alpha G\delta x \tag{2.10}$$

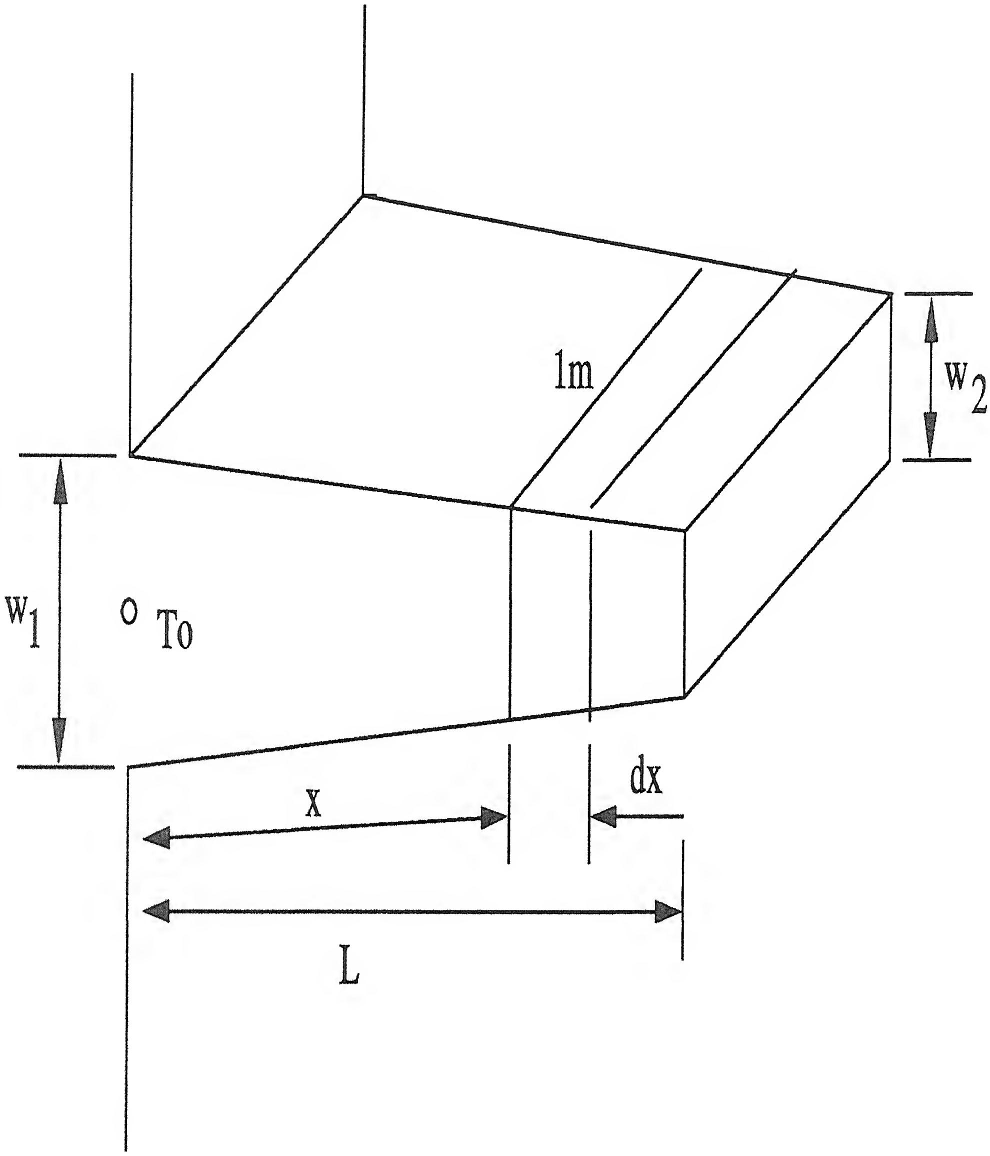

Figure 2.2: TRAPEZOIDAL FIN

No convective loss at the fin surface is included. For $\delta x \to 0.0$, the following differential equation must be satisfied for each point.

$$\frac{d}{dx}(k(W_1 - (W_1 - W_2)\frac{x}{L}\frac{dt}{dx}) = \epsilon\sigma(t^4 - \frac{\alpha G}{\epsilon\sigma}) \qquad (2.11)$$

The non-homogeneous term is recognized as the equilibrium temperature, or equivalent sink temperature, defined in equation. This temperature is the equilibrium temperature the surface would achieve if isolation in irradiation G. Thus with

$$t_s^4 = \frac{\alpha G}{\epsilon\sigma} \qquad (2.12)$$

The differential equation for the temperature distribution in the fin is

$$\frac{d}{dx}(k(W_1 - (W_1 - W_2)\frac{x}{L})\frac{dt}{dx}) - \epsilon\sigma(t^4 - t_s^4) = 0.0 \qquad (2.13)$$

Boundary conditions are

At $x = 0.0$

$$t = t_0 \qquad (2.14)$$

At $x = L$

$$\frac{dt}{dx} = 0.0 \qquad (2.15)$$

The solution of above equation equation is best discussed in terms of the

following dimensionless variables.

$$X = \frac{x}{L}$$

$$T = \frac{t}{t_0}$$

$$T_s = \frac{t_s}{t_0}$$

Introduction of these definitions in eq.(2.13) yields

$$\left(\frac{1.0}{\epsilon\sigma L^2 t_0^3}\right)\frac{d}{dT}\left(k(W_1 - (W_1 - W_2)X)\frac{dT}{dX}\right) - (T^4 - T_s^4) = 0.0 \qquad (2.16)$$

The dimensionless boundary conditions to be satisfied are

At $X = 0.0$

$$T = 1.0 \qquad (2.17)$$

At $X = 1.0$

$$\frac{dT}{dX} = 0.0 \qquad (2.18)$$

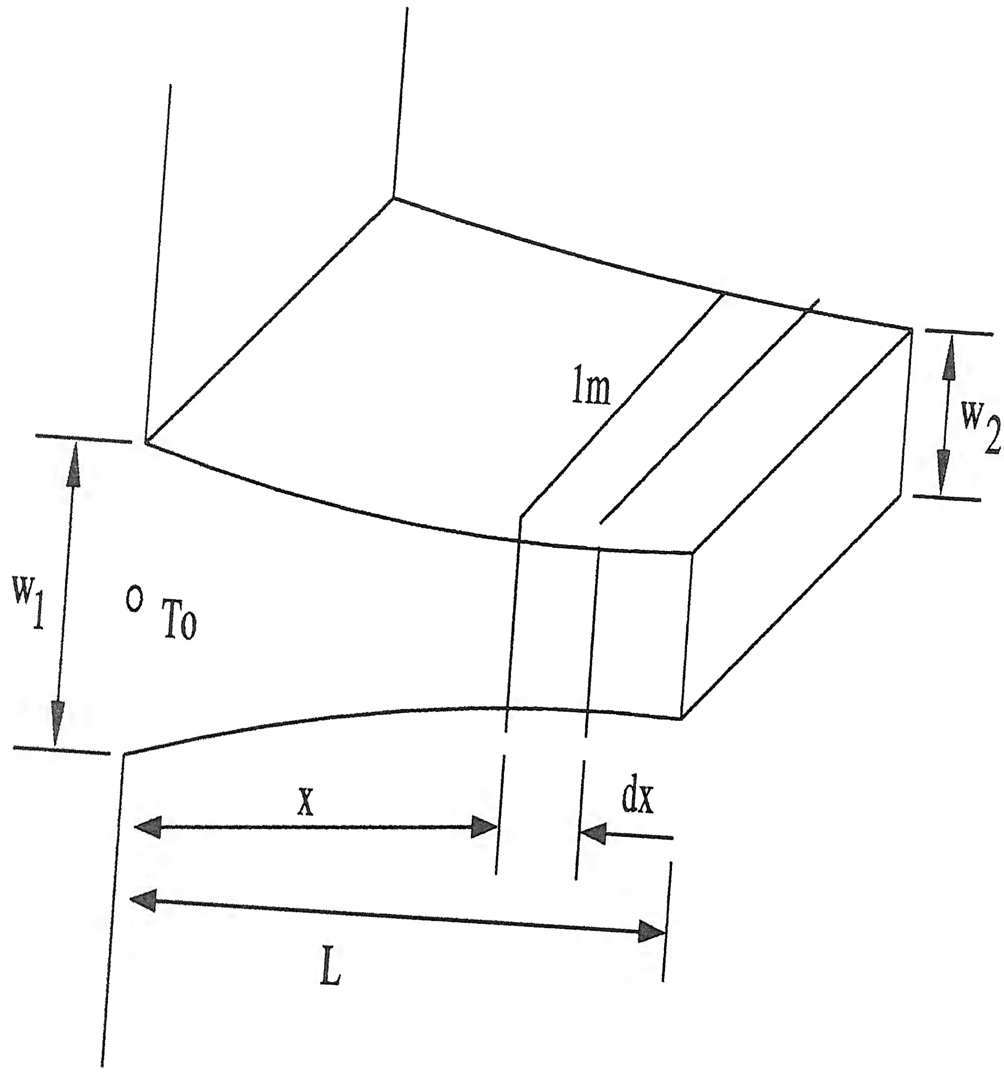

Figure 2.3: PARABOLIC FIN

2.2.3 FOR PARABOLIC FIN

If the conduction in the fin is taken to be one-dimensional,then the heat balance taken on an element δx in length yields the following relation (for a unit depth) for the parabolic fin of length L .If origin is taken at $x = 0.0$ and at the middle point of width and positive x axis is taken in the direction of fin length . The fin profile is of second degree parabola . See Fig.2.3.

'k ' may be also function of temperature. The actual functions of various fin materials used in this study are shown in Appendix 'B'.

Upper part of fin parabola

$$y = (a + bx + cx^2)$$

Lower part of fin parabola

$$y = -(a + bx + cx^2)$$

$$-2k(a+bx+cx^2)\frac{dt}{dx}\mid_x= -2k(a+bx+cx^2)\frac{dt}{dx}\mid_{x+\delta x} +\epsilon\sigma t^4\delta x - \alpha G\delta x \quad (2.19)$$

No convective loss at the fin surface is included. For $\delta x \rightarrow 0.0$, the following differential equation must be satisfied for each point.

$$\frac{d}{dx}(2k(a+bx+cx^2)\frac{dt}{dx}) = \epsilon\sigma(t^4 - \frac{\alpha G}{\epsilon\sigma}) \tag{2.20}$$

The non-homogeneous term is recognized as the equilibrium temperature, or equivalent sink temperature, defined in equation. This temperature is the equilibrium temperature the surface would achieve if isolation in irradiation G. Thus with

$$t_s^4 = \frac{\alpha G}{\epsilon\sigma} \tag{2.21}$$

The differential equation for the temperature distribution in the fin is

$$\frac{d}{dx}(2k(a+bx+cx^2)\frac{dt}{dx}) - \epsilon\sigma(t^4 - t_s^4) = 0.0 \tag{2.22}$$

Boundary conditions are

At $x = 0.0$

$$t = t_0 \tag{2.23}$$

At $x = L$

$$\frac{dt}{dx} = 0.0 \tag{2.24}$$

The solution of above equation equation is best discussed in terms of the following dimensionless variable.

$$X = \frac{x}{L}$$

$$T = \frac{t}{t_0}$$

$$T_s = \frac{t_s}{t_0}$$

Introduction of these definitions in eq. (2.22) yields

$$\left(\frac{1.0}{\epsilon\sigma L^2 t_0^3}\right)\frac{d}{dT}\left(2k(a + bXL + CX^2L^2)\frac{dT}{dX}\right) - (T^4 - T_s^4) = 0.0 \tag{2.25}$$

The boundary conditions to be satisfied are

At $X = 0.0$

$$T = 1.0 \tag{2.26}$$

At $X = 1.0$

$$\frac{dT}{dX} = 0.0 \tag{2.27}$$

2.3 METHOD OF SOLUTION

The finite-difference method is used to discretize the governing differential equations. The finite difference method enables one to integrate a differential

equation numerically by evaluating the values of the function at a discrete (finite) number of points . The origin of this method is Taylor series expansion which assumes that the function is smooth , that is continuous and differentiable. using central difference

$$\frac{dT}{dX} = \frac{T_{i+1} - T_{i-1}}{2\Delta X} + O(\Delta X)^2 \tag{2.28}$$

$$\frac{d^2T}{dX^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta X)^2} + O(\Delta X)^2 \tag{2.29}$$

The notation $O(\Delta X)^2$ means that in arriving above, term of order of $(\Delta X)^2$ and higher have been neglected. $O(\Delta X)^2$ is called the truncation error. The truncation error is the difference between the exact mathematical expression and numerical approximation.

2.3.1 DISCRETIZATION FOR RECTANGULAR FIN

Assuming

$$C_1 = \frac{W}{\epsilon\sigma L^2 t_0^3} \tag{2.30}$$

Governing equation

$$C_1 \frac{d}{dX}(k\frac{dT}{dX}) - (T^4 - T_s^4) = 0.0 \tag{2.31}$$

Boundary conditions

At $X = 0.0$.

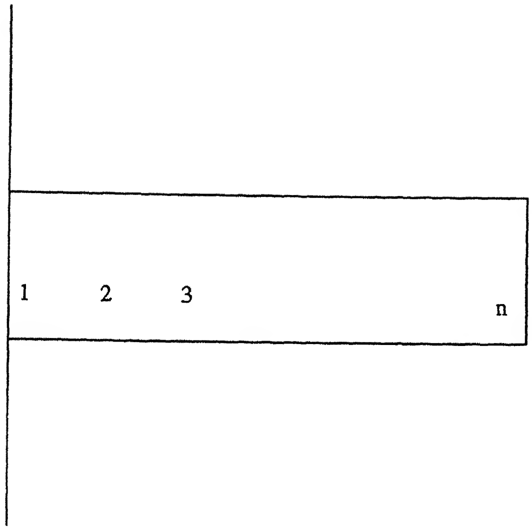

Figure 2.4: Grid point for Rectangular fin

$$T = 1.0 \tag{2.32}$$

At$X = L$

$$\frac{dT}{dX} = 0.0 \tag{2.33}$$

The above equation can be written as

$$C_1 \frac{d^2T}{dX^2} + \frac{dT}{dX} C_1 \frac{dk}{dX} - (T_o^3 T - T_s^4) = 0.0 \tag{2.34}$$

T_o indicate non-dimensional temperature in previous iteration.

For $i = 1$

$$T_1 = 1.0 \tag{2.35}$$

k_i is the thermal conductivity of material at i^{th} point at T_{oi} .

For $i = 2$

$$C_1 k_i \frac{T_{i+1} + 1.0 - 2T_i}{\Delta X^2} + C_1 (\frac{T_{i+1} - 1.0}{2\Delta X})(\frac{k_{i+1} - k_{i-1}}{2\Delta X}) - (T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.36}$$

$$T_i(-\frac{2C_1 k_i}{\Delta X^2} - T_{oi}^3) + T_{i+1}(\frac{C_1 k_i}{\Delta X^2} + \frac{C_1(k_{i+1} - k_{i-1})}{2\Delta X^2})$$

$$= -C_1(\frac{4k_i - k_{i+1} + k_{i-1}}{4\Delta X^2}) - T_s^4 \tag{2.37}$$

For $i = 3, ..., n-2, n-1$

$$C_1 k_i \frac{T_{i+1} + T_{i-1} - 2T_i}{\Delta X^2} + C_1 (\frac{T_{i+1} - T_{i-1}}{2\Delta X})(\frac{k_{i+1} - k_{i-1}}{2\Delta X}) - (T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.38}$$

$$T_{i-1}(\frac{C_1(4k_i - k_{i+1} + k_{i-1})}{4\Delta X^2}) + T_i(-\frac{2C_1 k_i}{\Delta X^2} - T_{oi}^3) + T_{i+1}(\frac{C_1 k_i}{\Delta X^2} + \frac{C_1(k_{i+1} - k_{i-1})}{2\Delta X^2})$$

$$= -T_s^4 \tag{2.39}$$

For $i = n$ using image-point technique, eq.(2.39) takes the form

$$T_{i-1}(\frac{2C_1k_i}{\Delta X^2}) + T_i(-\frac{2C_ik_i}{\Delta X^2} - T_{oi}^3) = -T_S^4 \tag{2.40}$$

2.3.2 DISCRETIZATION FOR TRAPEZOIDAL FIN

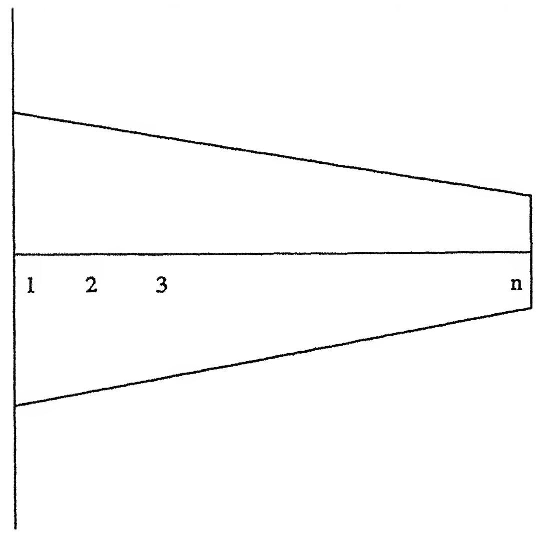

Figure 2.5: Grid points for Trapezoidal fin

Assuming

$$C_2 = \frac{1.0}{\epsilon\sigma t_0^3 L^2} \tag{2.41}$$

$$kC_2((W_1-(W_1-W_2)X)\frac{d^2T}{dX^2}-kC_2(W_1-W_2)\frac{dT}{dX}+C_2(W_1-(W_1-W_2)X)\frac{dT}{dX}\frac{dk}{dX}$$

$$-T_o^3T_i - T_s^4 = 0.0 \tag{2.42}$$

Assuming

$$S_i = W_1 - (W_1 - W_2)X_i \tag{2.43}$$

T_o indicates non-dimensional temperature in previous iteration.

For $i = 1$

$$T_1 = 1.0 \tag{2.44}$$

k_i is the thermal conductivity of material at i^{th} point at T_{oi} .

For $i = 2$

$C_2 k_i S_i \frac{T_{i+1}+1.0-2T_i}{\Delta X^2} - k_i C_2 (W_1 - W_2) \frac{T_{i+1}-1.0}{2\Delta X}$

$$+C_2 S_i (\frac{T_{i+1} - T_{i-1}}{2\Delta X})(\frac{k_{i+1} - k_{i-1}}{2\Delta X}) - (T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.45}$$

$$T_i(-\frac{2C_2 k_i S_i}{\Delta X^2} - T_{oi}^3) + T_{i+1}(\frac{S_i C_2 k_i}{\Delta X^2} - \frac{C_2 k_i (W_1 - W_2)}{2\Delta X} + C_2 S_i \frac{(k_{i+1} - k_{i-1})}{4\Delta X^2})$$

$$= -T_s^4 - (\frac{S_i C_2 k_i}{\Delta X^2} + C_2 \frac{k_i (W_1 - W_2)}{2\Delta X} - C_2 S_i \frac{k_{i+1} - k_{i-1}}{4\Delta X^2}) \tag{2.46}$$

For $i = 3, ..., n-2, n-1$

$C_2 k_i S_i \frac{T_{i+1}+T_{i-1}-2T_i}{\Delta X^2} - k_i C_2 (W_1 - W_2) \frac{T_{i+1}-T_{i-1}}{2\Delta X}$

$$+C_2 S_i (\frac{T_{i+1} - T_{i-1}}{2\Delta X})(\frac{k_{i+1} - k_{i-1}}{2\Delta X}) - (T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.47}$$

$$T_i(-\frac{2C_2k_iS_i}{\Delta X^2} - T_{oi}^3) + T_{i+1}(\frac{S_iC_2k_i}{\Delta X^2} - \frac{C_2k_i(W_1 - W_2)}{2\Delta X} + C_2S_i\frac{(k_{i+1} - k_{i-1})}{4\Delta X^2})$$

$$+T_{i-1}(\frac{S_iC_2k_i}{\Delta X^2} + C_2\frac{k_i(W_1 - W_2)}{2\Delta X} - C_2S_i\frac{k_{i+1} - k_{i-1}}{4\Delta X^2}) = -T_s^4 \qquad (2.48)$$

For $i = n$, using image-point technique,

$$T_{i-1} = T_{i+1} \qquad (2.49)$$

Putting eq.(2.49) in eq. (2.48),

$$T_{i-1}(\frac{2C_2S_ik_i}{\Delta X^2}) + T_i(-\frac{2C_2S_ik_i}{\Delta X^2} - T_{oi}^3) = -T_s^4 \qquad (2.50)$$

2.3.3 DISCRETIZATION FOR PARABOLIC FIN

Assuming

$$C_2 = \frac{1.0}{\epsilon\sigma L^2 t_0^3} \qquad (2.51)$$

Governing equation

$$C_2\frac{d}{dX}(k2(a + bXL + CX^2L^2)\frac{dT}{dX}) - (T^4 - T_s^4) = 0.0 \qquad (2.52)$$

Boundary conditions

At $X = 0.0$.

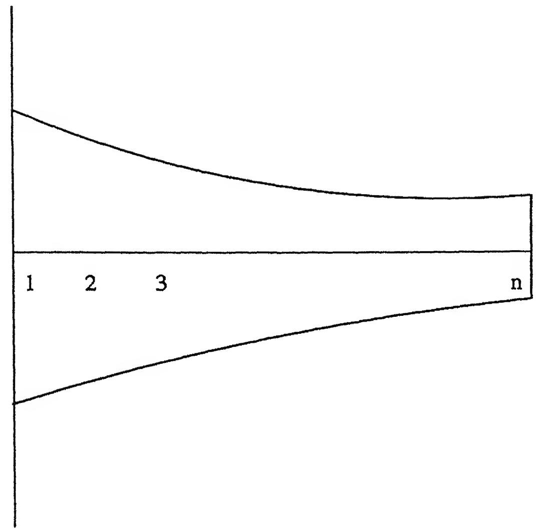

Figure 2.6: Grid points for Parabolic fin

$$T = 1.0 \tag{2.53}$$

At$X = L$

$$\frac{dT}{dX} = 0.0 \tag{2.54}$$

The above equation can be written as

$$C_2 k \frac{d^2T}{dX^2}(a+bXL+CX^2L^2)+\frac{dT}{dX}C_2(a+bXL+CX^2L^2)\frac{dk}{dX}+(bL+2XL^2)C_2 k\frac{dT}{dX}$$

$$-(T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.55}$$

Assuming

$$S1_i = 2(a + bXL + cX^2L^2) \tag{2.56}$$

T_o indicates non-dimensional temperature in previous iteration .

For $i = 1$

$$T_1 = 1.0 \tag{2.57}$$

k_i is the thermal conductivity of material at i^{th} point at T_{oi} .

For $i = 2$

$C_2 k_i S1_i \frac{T_{i+1}+1.0-2T_i}{\Delta X^2} + k_i C_2(bL + 2cXL^2)\frac{T_{i+1}-1.0}{2\Delta X}$

$$+C_2 S1_i(\frac{T_{i+1} - 1.0}{2\Delta X})(\frac{k_{i+1} - k_{i-1}}{2\Delta X}) - (T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.58}$$

$$T_i(-\frac{2C_2 k_i S1_i}{\Delta X^2} - T_{oi}^3) + T_{i+1}(\frac{S1_i C_2 k_i}{\Delta X^2} + \frac{C_2 k_i(bL + 2cXL^2)}{2\Delta X} + C_2 S1_i \frac{(k_{i+1} - k_{i-1})}{4\Delta X^2})$$

$$= -T_s^4 - (\frac{S1_i C_2 k_i}{\Delta X^2} - C_2 \frac{k_i(bX + 2cXL^2)}{2\Delta X} - C_2 S1_i \frac{k_{i+1} - k_{i-1}}{4\Delta X^2}) \tag{2.59}$$

For $i = 3, ..., n-2, n-1$

$C_2 k_i S1_i \frac{T_{i+1}+T_{i-1}-2T_i}{\Delta X^2} + k_i C_2(bL + 2cXL^2)\frac{T_{i+1}-T_{i-1}}{2\Delta X}$

$$+C_2 S_i(\frac{T_{i+1} - T_{i-1}}{2\Delta X})(\frac{k_{i+1} - k_{i-1}}{2\Delta X}) - (T_{oi}^3 T_i - T_s^4) = 0.0 \tag{2.60}$$

$$T_i(-\frac{2C_2 k_i S1_i}{\Delta X^2} - T_{oi}^3) + T_{i+1}(\frac{S1_i C_2 k_i}{\Delta X^2} + \frac{C_2 k_i(bL + 2cXL^2)}{2\Delta X} + C_2 S1_i \frac{(k_{i+1} - k_{i-1})}{4\Delta X^2})$$

$$+T_{i-1}(\frac{S1_iC_2k_i}{\Delta X^2} - C_2\frac{k_i(bX + 2cXL^2)}{2\Delta X} - C_2S1_i\frac{k_{i+1} - k_{i-1}}{4\Delta X^2}) = -T_s^4 \quad (2.61)$$

For $i = n$, using image-point technique , eq.(2.61) takes form

$$T_{i-1}(\frac{2C_2S1_ik_i}{\Delta X^2}) + T_i(-\frac{2C_2S1_ik_i}{\Delta X^2} - T_{oi}^3) = -T_s^4 \quad (2.62)$$

2.4 CALCULATION OF HEAT TRANSFER AND EFFICIENCY OF THE FIN

The complete solution of the nonlinear equation here can be obtained the by an iterative method. Rather than seek the distribution , the principal item of interest is the heat dissipated by the fin .

$$q = -k_1W(\frac{dt}{dx})_{x=0.0} \quad (2.63)$$

W in case of trapezoidal fin

$$W = W_1 \quad (2.64)$$

W in case of parabolic fin of profile

For upper part

$$y = (a + bx + cx^2)$$

For lower part

$$y = -(a + bx + cx^2)$$

$$W = 2a \tag{2.65}$$

In form of non dimensionless variables the q

$$q = -\frac{k_1 W t_0}{L}\left(\frac{dT}{dX}\right)_{i=1} \tag{2.66}$$

The heat dissipated is best expressed in term of fin efficiency, which is the ratio of q to the heat that would be dissipated if the whole fin is maintained at base temperature.

$$efficiency = \frac{q}{(\epsilon\sigma t_0^4 - \alpha G)L} \tag{2.67}$$

Eq.(2.67) can also be written in other form also

$$efficiency = \frac{q}{\epsilon\sigma L t_0^4(1 - T_s^4)} \tag{2.68}$$

$$efficiency = -\frac{1.0}{\lambda(1-T_s^4)}(\frac{dT}{dX})_{i=1} \tag{2.69}$$

Where

$$\lambda = \frac{\epsilon\sigma t_0^3 L^2}{kW} \tag{2.70}$$

2.5 METHOD OF SOLUTION (along with flow chart)

The resulting equation after discretization are solved by the well known Tri-diagonal matrix algorithm (TDMA). The flow chart of the overall solution procedure is shown in Fig.2.7. A grid independence test has been performed for each type of fin. It is found that 20 grid points are sufficient.

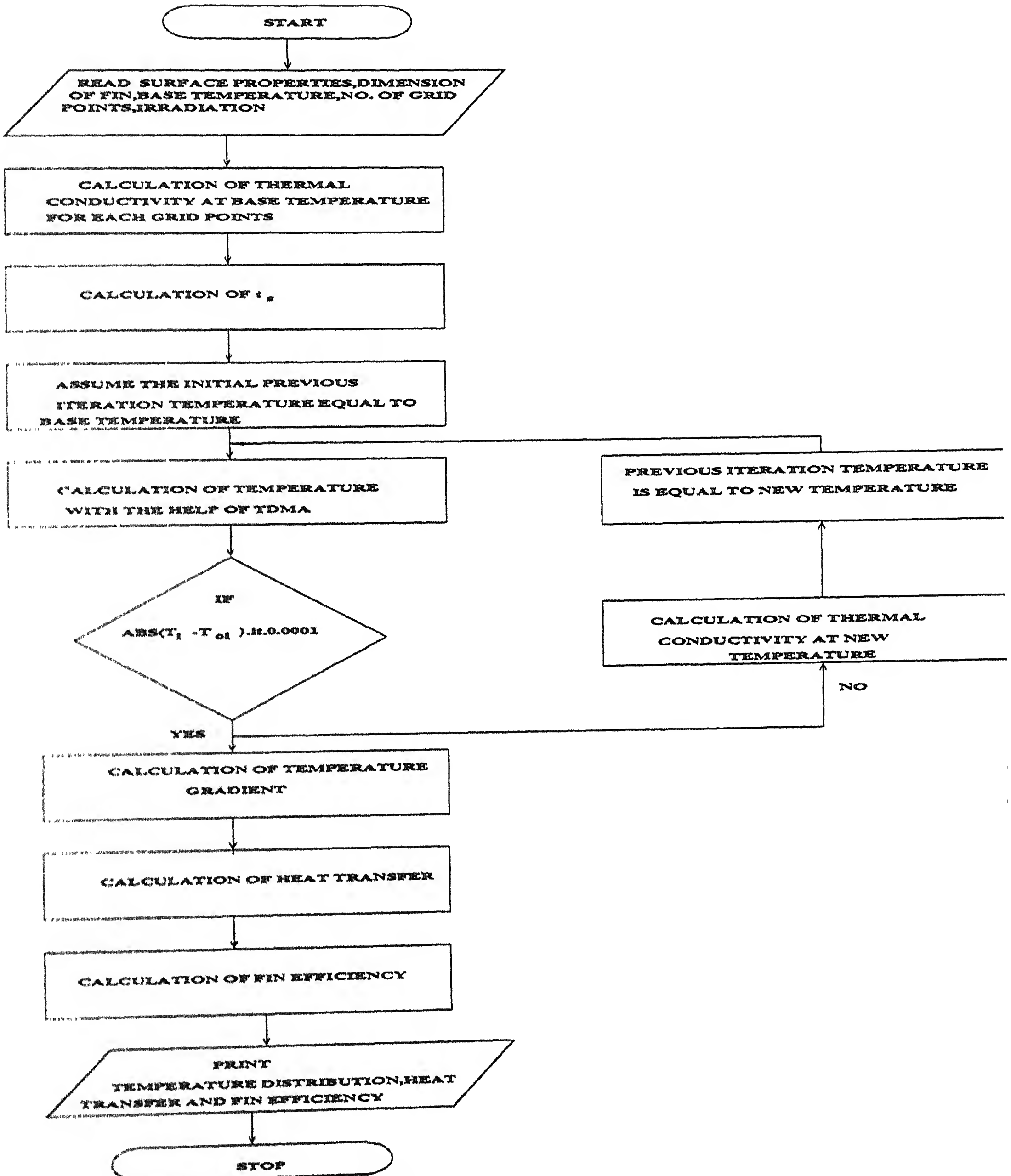

Figure 2.7: FLOW CHART

Chapter 3

RESULTS AND DISCUSSION

Here the result are given in mainly two parts, that is, for constant conductivity and variable conductivity. For all the results presented here, $\epsilon = 0.5$, $\alpha = 0.1$, $G = 315.4\frac{W}{m^2}$.

3.1 CONSTANT CONDUCTIVITY OF FIN MATERIALS

3.1.1 RECTANGULAR FIN

TEMPERATURE DISTRIBUTION

The temperature of rectangular fin of steel of constant conductivity($k =$ 45 W/mK).See Fig. 3.1

Taking

$$W = 0.000375m$$

$$L = 0.075m$$

COMPARISON WITH 'NASA' RESULT

The Efficiency Vs $\sqrt{\lambda}$ for rectangular is compared with the S.Lieblein,NASA Tech. Note D-196, National Aeronautics and Space Administration ,Washington, D.C.1959. and is found very satisfactory (Fig.3.3). It is to be noted that the NASA results are available only in the graphical form in Chapman (1989) . The NASA results are shown in Fig.3.2.

3.1.2 TRAPEZOIDAL FIN

TEMPERATURE DISTRIBUTION

Temperature of trapezoidal fin of steel of constant conductivity ($k = 45$ W/mK). See Fig. 3.4.

Using

$$W_1 = 0.000375m$$

$$W_2 = \frac{W_1}{2.0}$$

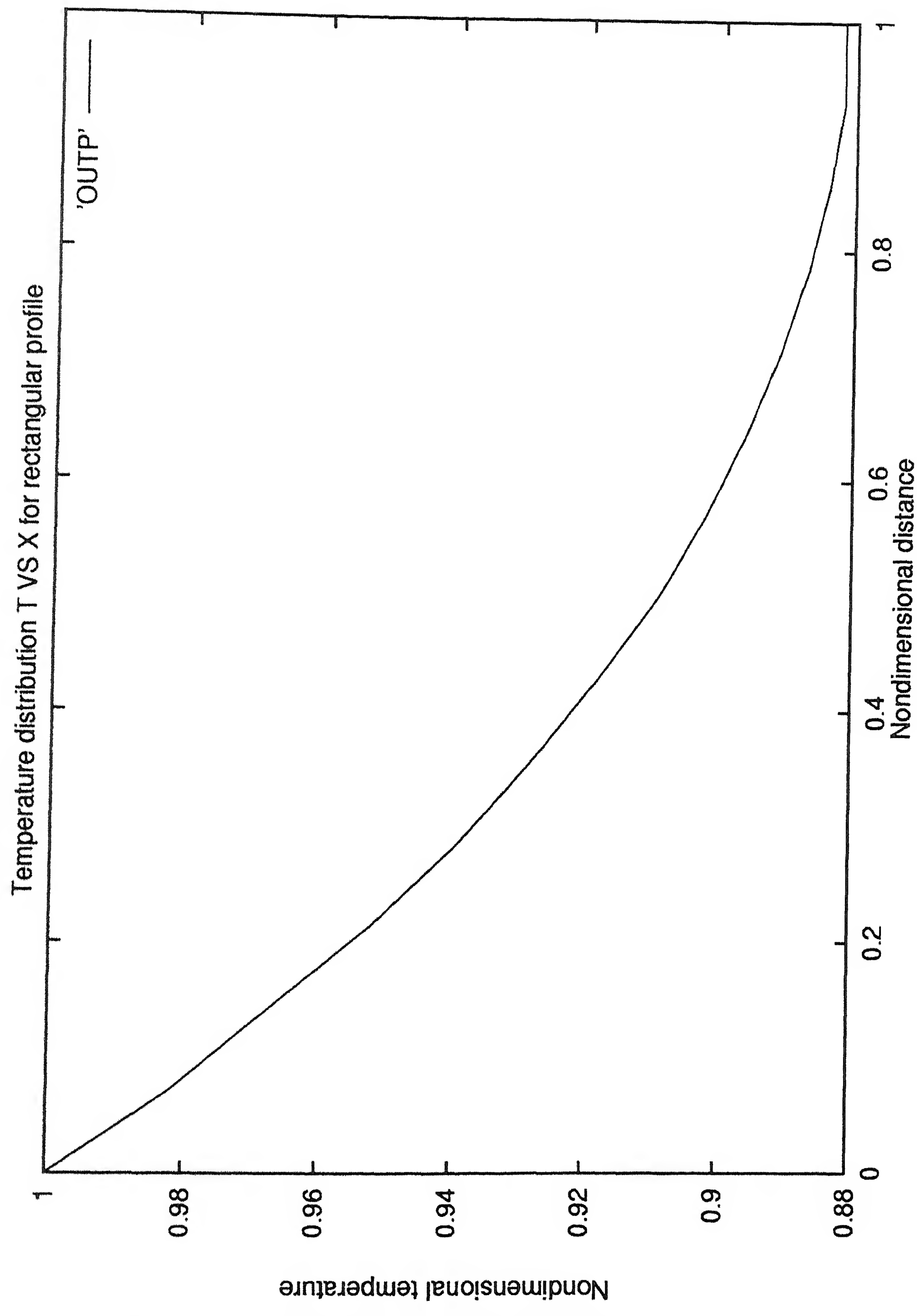

Figure 3.1: Temperature distribution of rectangular fin of constant conductivity

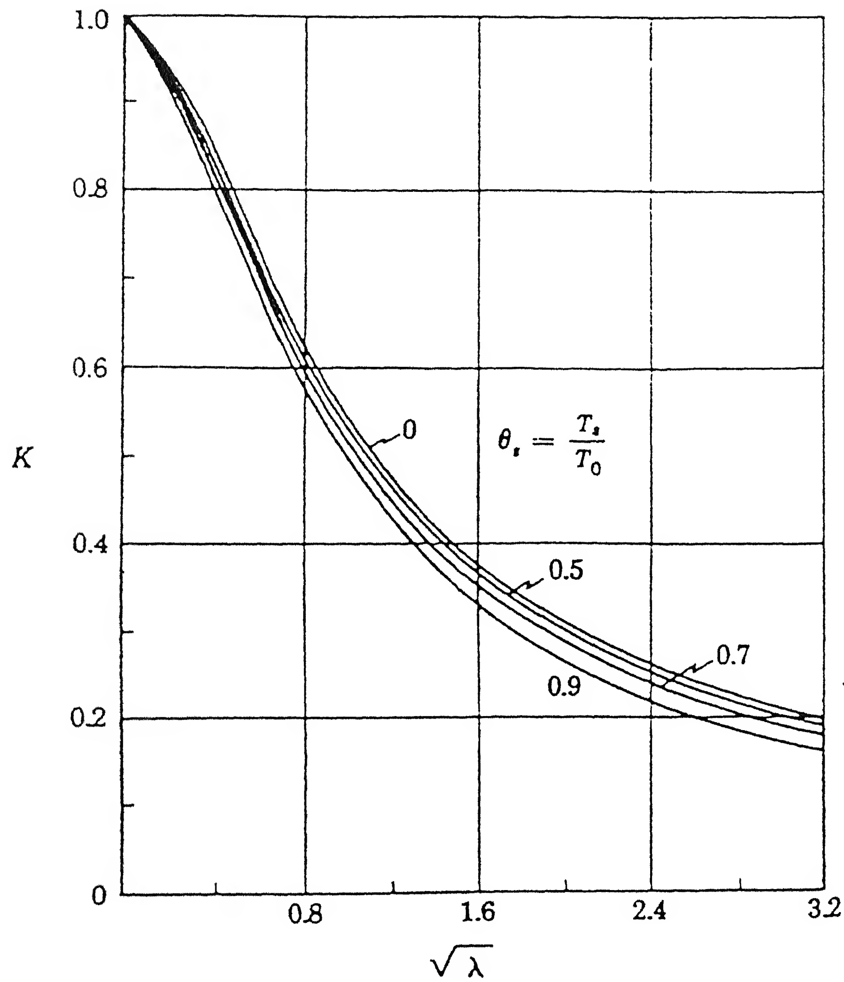

Figure 3.2: Actual NASA Result

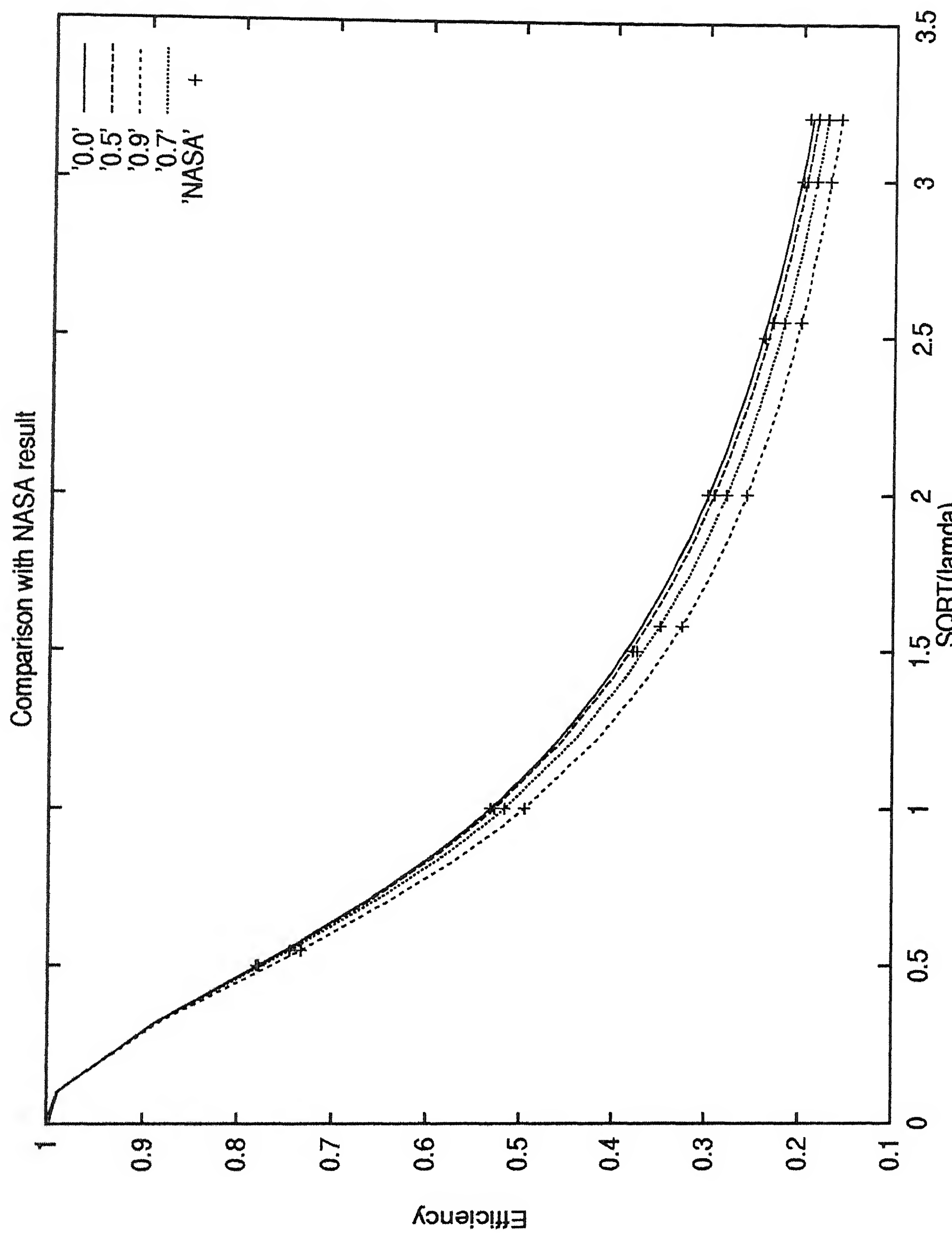

Figure 3.3: Efficiency Vs $\sqrt{\lambda}$ for rectangular fin of constant conductivity '0.0' indicates $\frac{t_s}{t_0} = 0.0$, '0.5 'indicates $\frac{t_s}{t_0} = 0.5$, '0.7' indicates $\frac{t_s}{t_0} = 0.7$ '0.9 'indicates $\frac{t_s}{t_0} = 0.9$, 'NASA' indicates original NASA result

$$L = 0.1125m$$

EFFICIENCY VS $\sqrt{\lambda}$

Here $W2 = \frac{W_1}{2}$

Here λ is

$$\lambda = \frac{\epsilon \sigma t_0^3 L^2}{k W_1} \tag{3.1}$$

See Fig. 3.5.

3.1.3 PARABOLIC FIN

TEMPERATURE DISTRIBUTION

For a parabolic fin, the graph of T Vs X is given here for steel of $k = 45$ W/mK. See fig. 3.6.

Using

$$a = 0.0001875$$

$$b = -0.001125$$

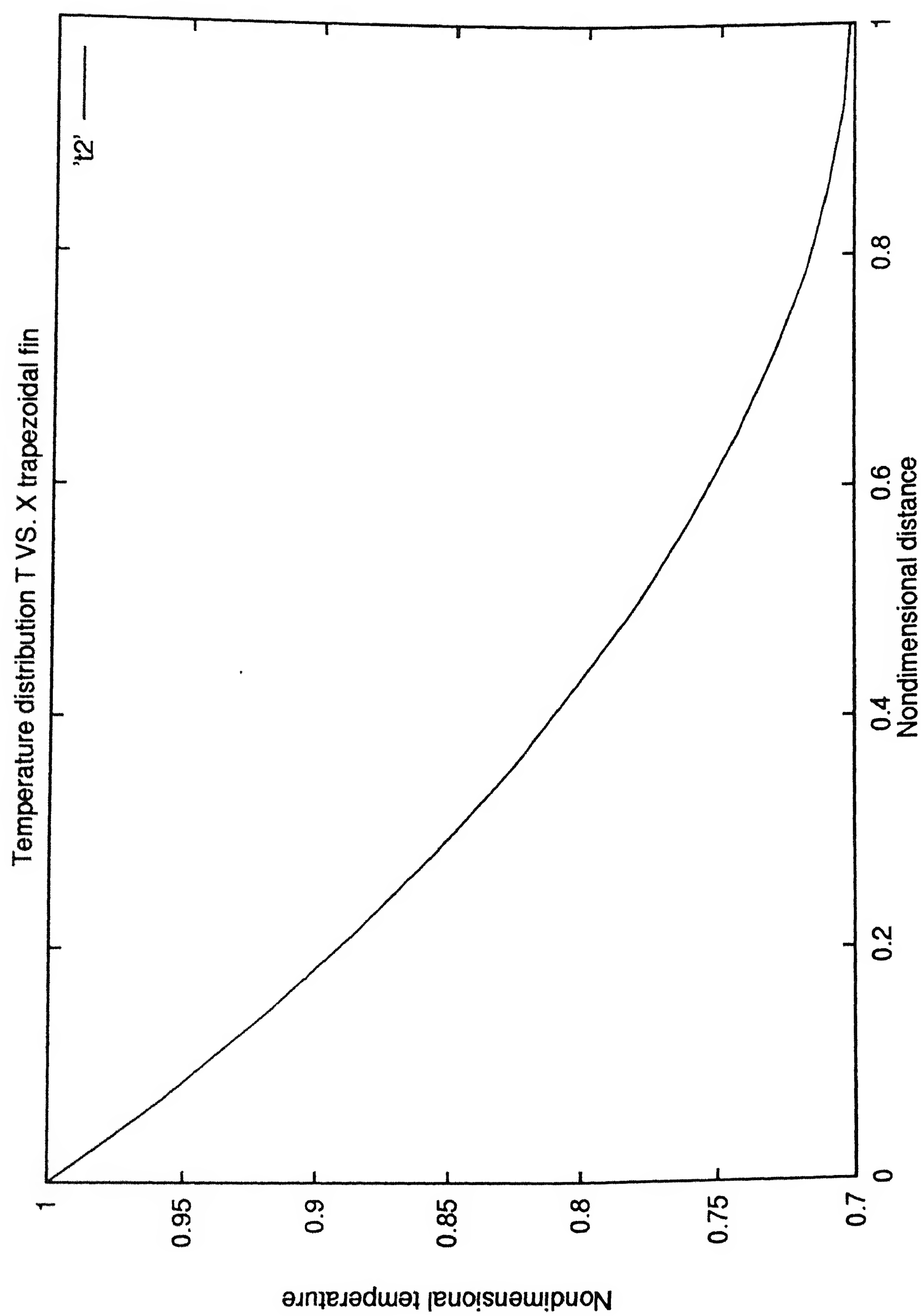

Figure 3.4: T Vs X for Trapezoidal fin for constant conductivity

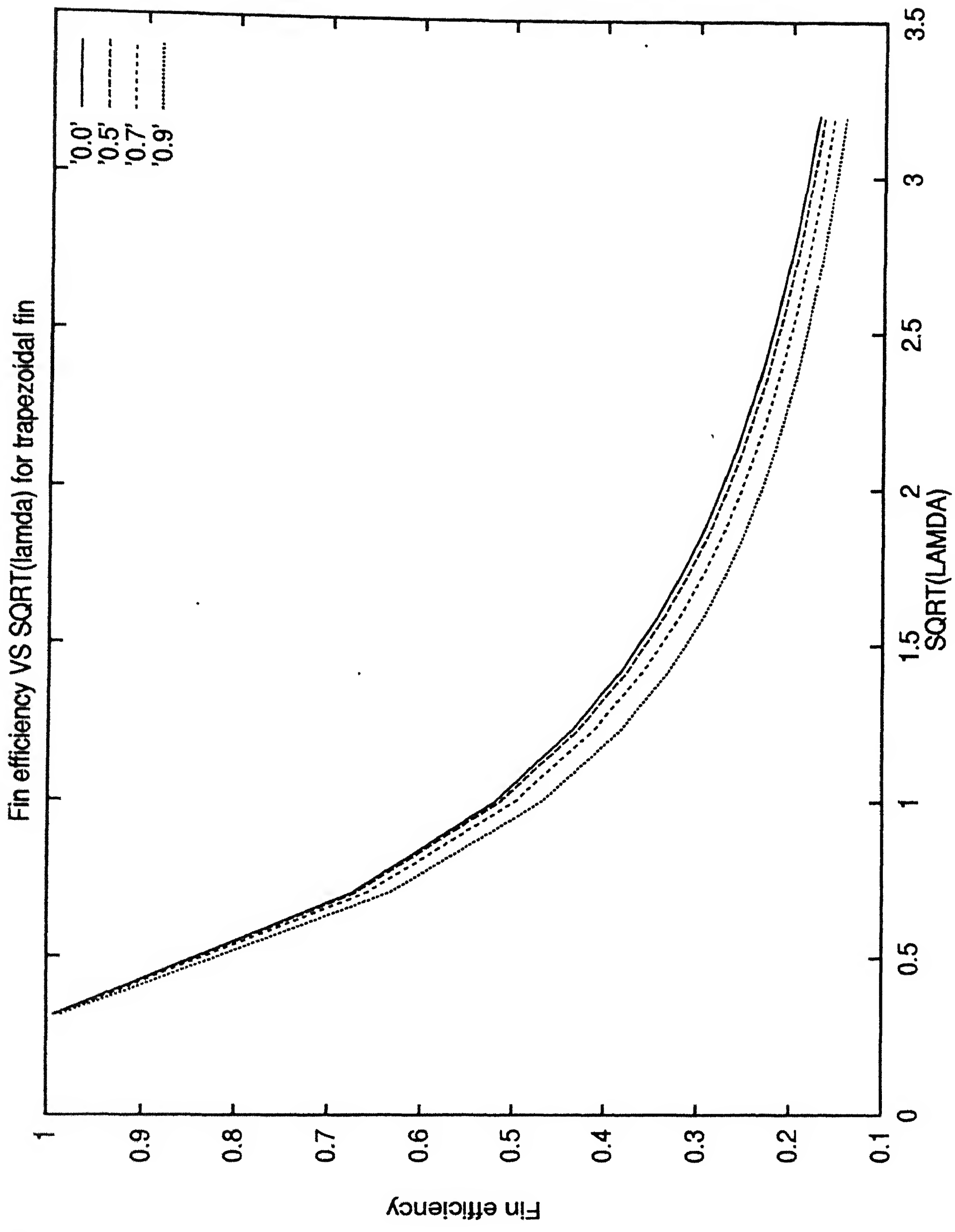

Figure 3.5: Efficiency Vs $\sqrt{\lambda}$ for Trapezoidal fin of constant conductivity '0.0' indicates $\frac{t_s}{t_0} = 0.0$, '0.5 'indicates $\frac{t_s}{t_0} = 0.5$, '0.7' indicates $\frac{t_s}{t_0} = 0.7$ '0.9 'indicates $\frac{t_s}{t_0} = 0.9$,

$$c = 0.000005$$

$$L = 0.1m$$

EFFICIENCY VS $\sqrt{\lambda}$

Using

$$a = 0.0001875$$

$$b = -0.001125$$

$$c = 0.000005$$

$$L = 0.1m$$

Here λ is

$$\lambda = \frac{\epsilon \sigma t_0^3 L^2}{2ka} \tag{3.2}$$

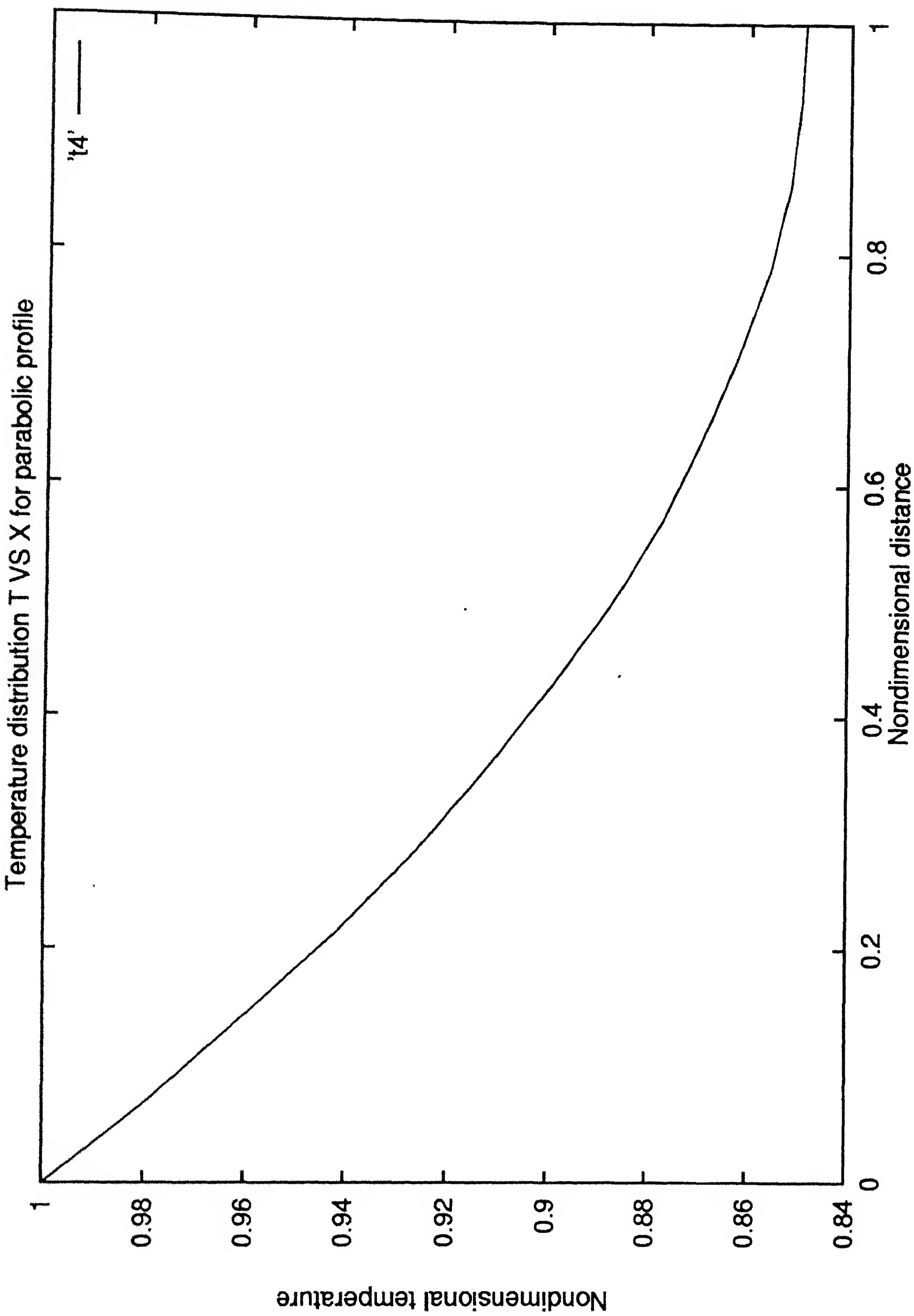

Figure 3.6: T Vs X for parabolic fin for constant conductivity

See Fig. 3.7.

3.2 VARIABLE CONDUCTIVITY OF FIN MATERIAL

3.2.1 RECTANGULAR FIN

TEMPERATURE DISTRIBUTION

The temperature distribution of Rectangular fin is drawn taking copper as the fin material. If the base temperature is low, then there is almost no variation in temperature distribution with respect to constant and variable conductivity . Therefore ,the base temperature is taken as 673K. See Fig. 3.8.

Taking

$$W = 0.000375m$$

$$L = 0.075m$$

EFFICIENCY VS $\sqrt{\lambda}$

Base temperature is taken as 323K.

For carbon steel

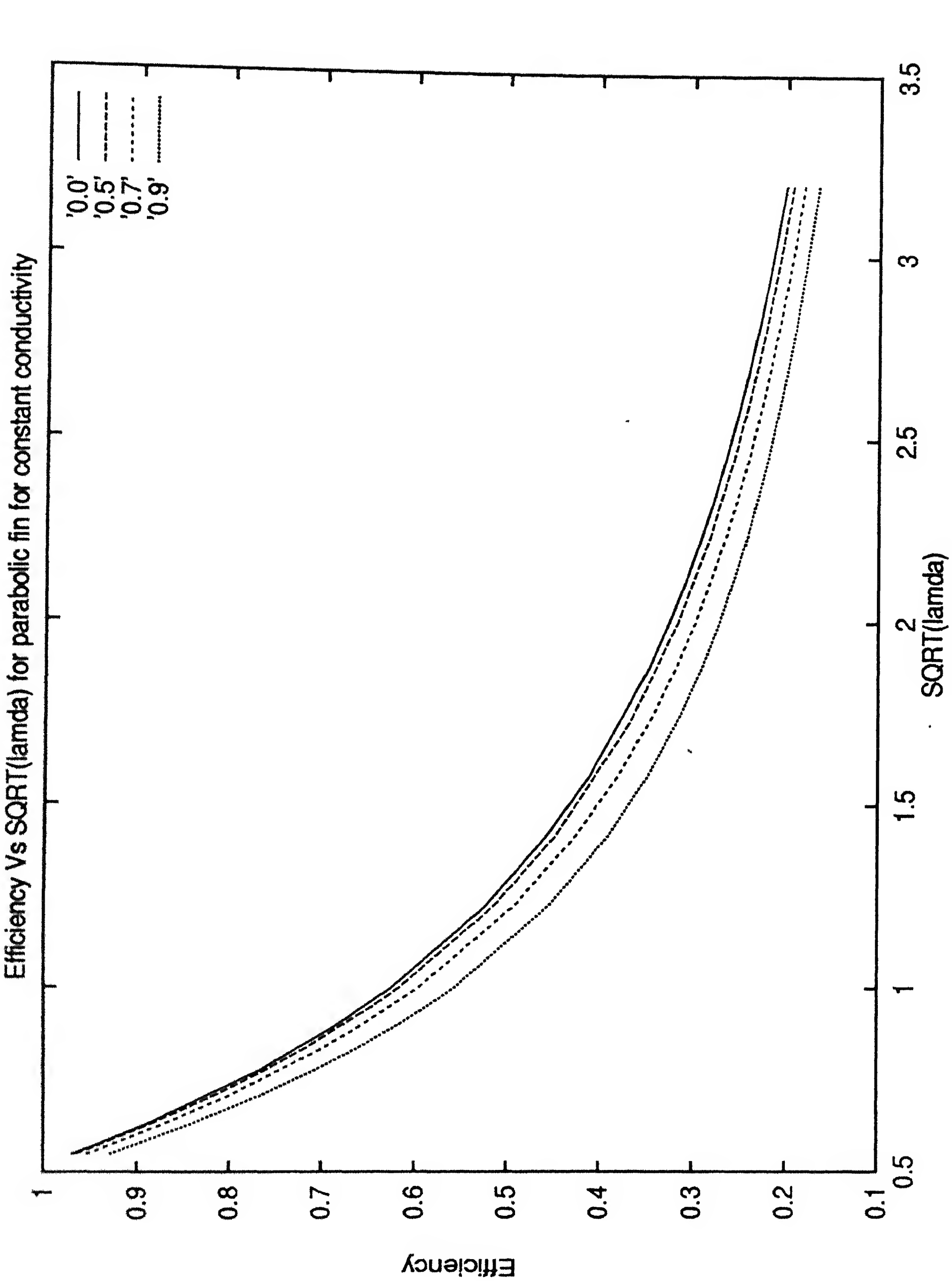

Figure 3.7: Efficiency Vs $\sqrt{\lambda}$ for parabolic fin of constant conductivity '0.0' indicates $\frac{t_s}{t_0} = 0.0$, '0.5 'indicates $\frac{t_s}{t_0} = 0.5$, '0.7' indicates $\frac{t_s}{t_0} = 0.7$ '0.9 'indicates $\frac{t_s}{t_0} = 0.9$,

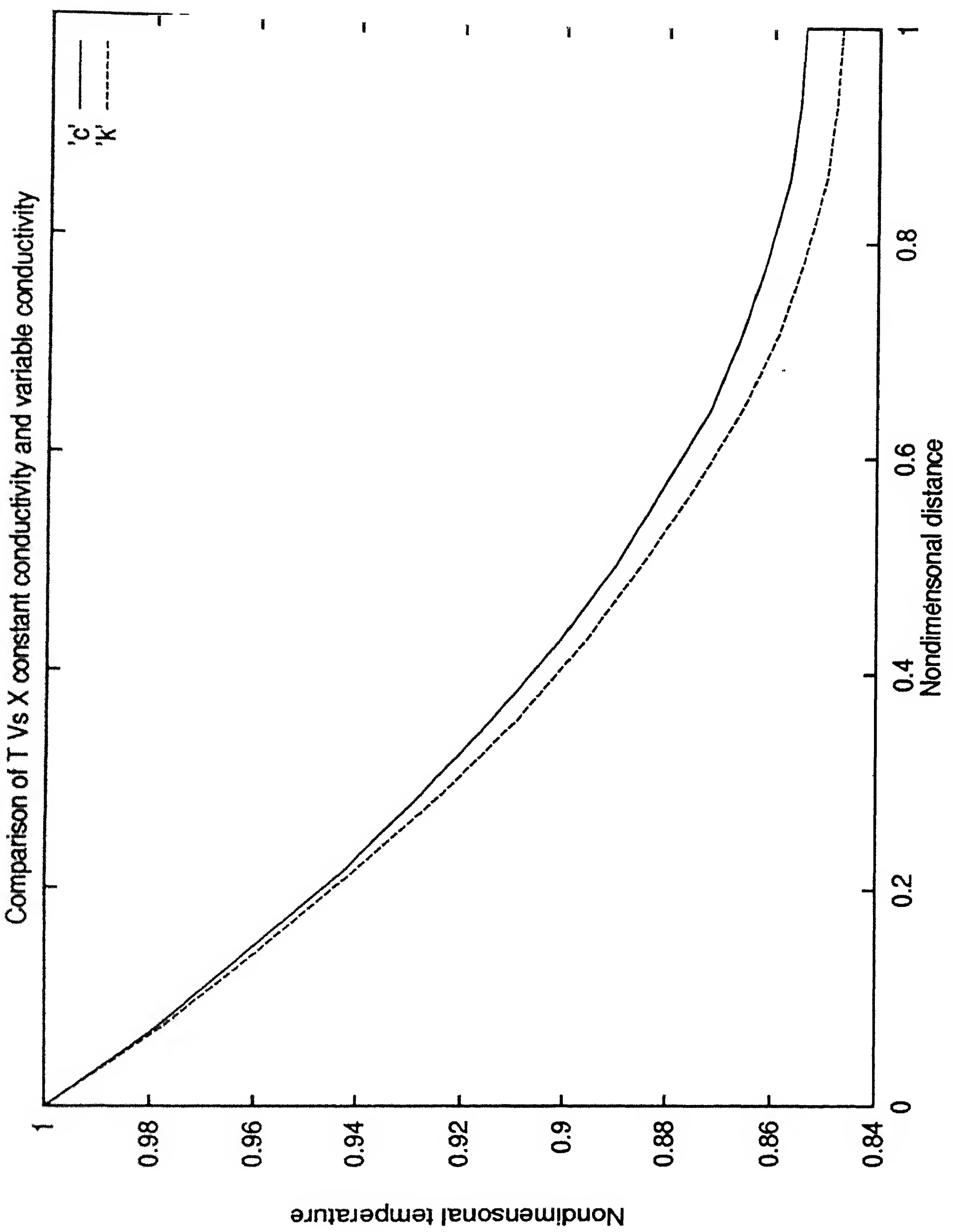

Figure 3.8: Comparison of T Vs X for constant conductivity and variable conductivity for rectangular fin of copper
'c' indicates constant conductivity. 'k' indicates variable conductivity

$$\lambda = \frac{\epsilon \sigma t_0^3 L^2}{k_1 W} \tag{3.3}$$

k_1 is Thermal conductivity at base temperature.

See Fig. 3.9.

3.2.2 TRAPEZOIDAL FIN

TEMPERATURE DISTRIBUTION

The temperature distribution of Trapezoidal fin is drawn taking copper as the fin material. If the base temperature is low, then there is almost no variation in temperature distribution with respect to constant and variable conductivity . Therefore ,the base temperature is taken as 673K. See Fig. 3.10 .

Using

$$W_1 = 0.000375m$$

$$W_2 = \frac{W_1}{2.0}$$

$$L = 0.1125m$$

EFFICIENCY VS $\sqrt{\lambda}$

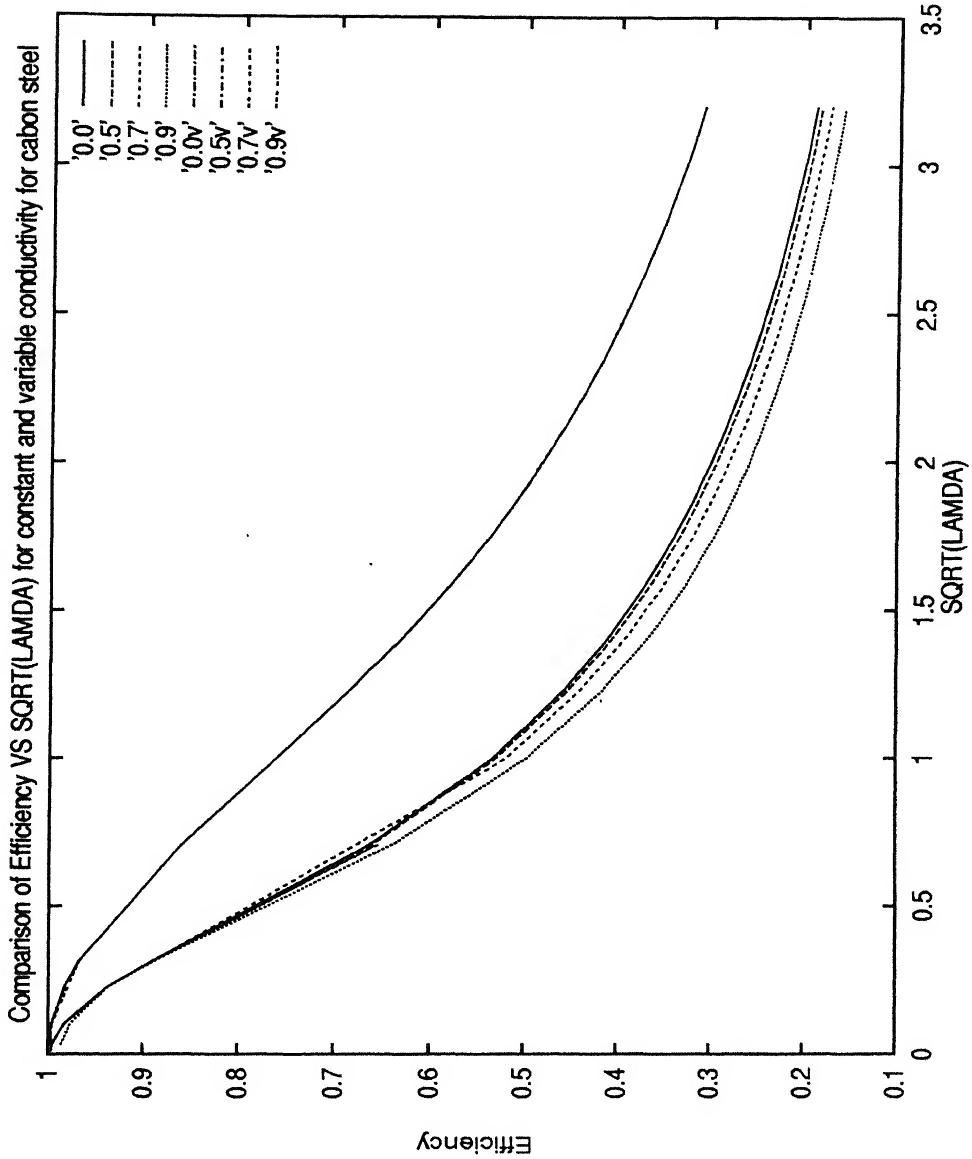

Figure 3.9: Comparison of Efficiency Vs $\sqrt{\lambda}$ for constant conductivity and variable conductivity for rectangular fin of carbon steel
'0.0' indicates $\frac{t_s}{t_0} = 0.0$, '0.5 'indicates $\frac{t_s}{t_0} = 0.5$, '0.7' indicates $\frac{t_s}{t_0} = 0.7$
'0.9 'indicates $\frac{t_s}{t_0} = 0.9$,
For variable conductivity material '0.0v' indicates $\frac{t_s}{t_0} = 0.0$,
'0.5v' indicates $\frac{t_s}{t_0} = 0.5$, '0.7v' indicates $\frac{t_s}{t_0} = 0.7$, '0.9v' indicates $\frac{t_s}{t_0} = 0.9$.

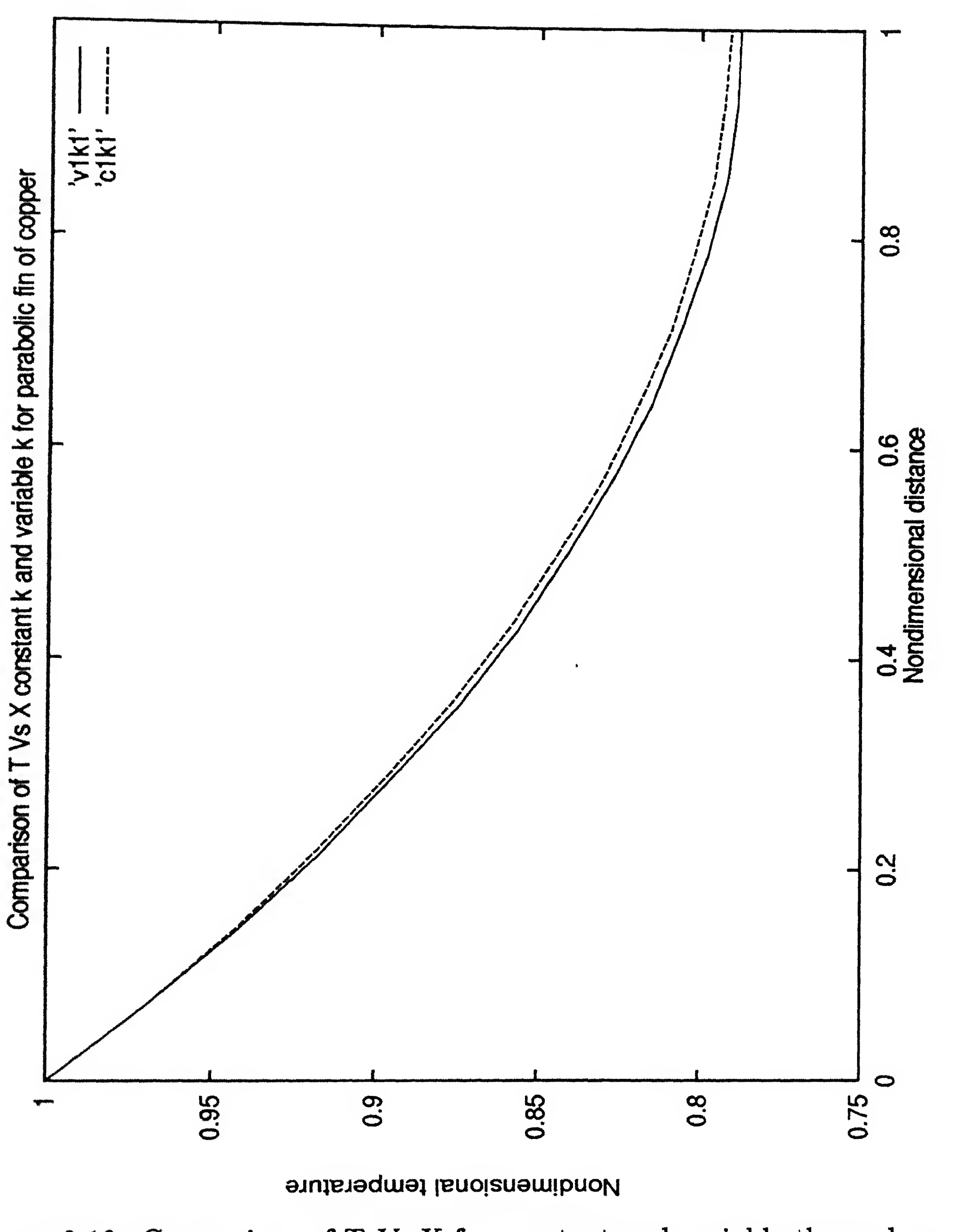

Figure 3.10: Comparison of T Vs X for constant and variable thermal conductivity for trapezoidal fin of copper
'c1k1' indicates constant conductivity. 'c2k2' indicates variable conductivity

For carbon steel for a particular case $W_2 = \frac{W_1}{2}$

Base temperature is taken as 323K

$$\lambda = \frac{\epsilon \sigma t_0^3 L^2}{k_1 W_1} \tag{3.4}$$

k_1 is the the thermal conductivity of carbon steel at base temperature. See Fig. 3.11 .

3.2.3 PARABOLIC FIN

TEMPERATURE DISTRIBUTION

The temperature distribution of Parabolic fin is drawn taking copper as the fin material. If the base temperature is low, then there is almost no variation in temperature distribution with respect to constant and variable conductivity . Therefore ,the base temperature is taken as 673K. See Fig. 3.12 .

Using

$$a = 0.0001875$$

$$b = -0.001125$$

$$c = 0.000005$$

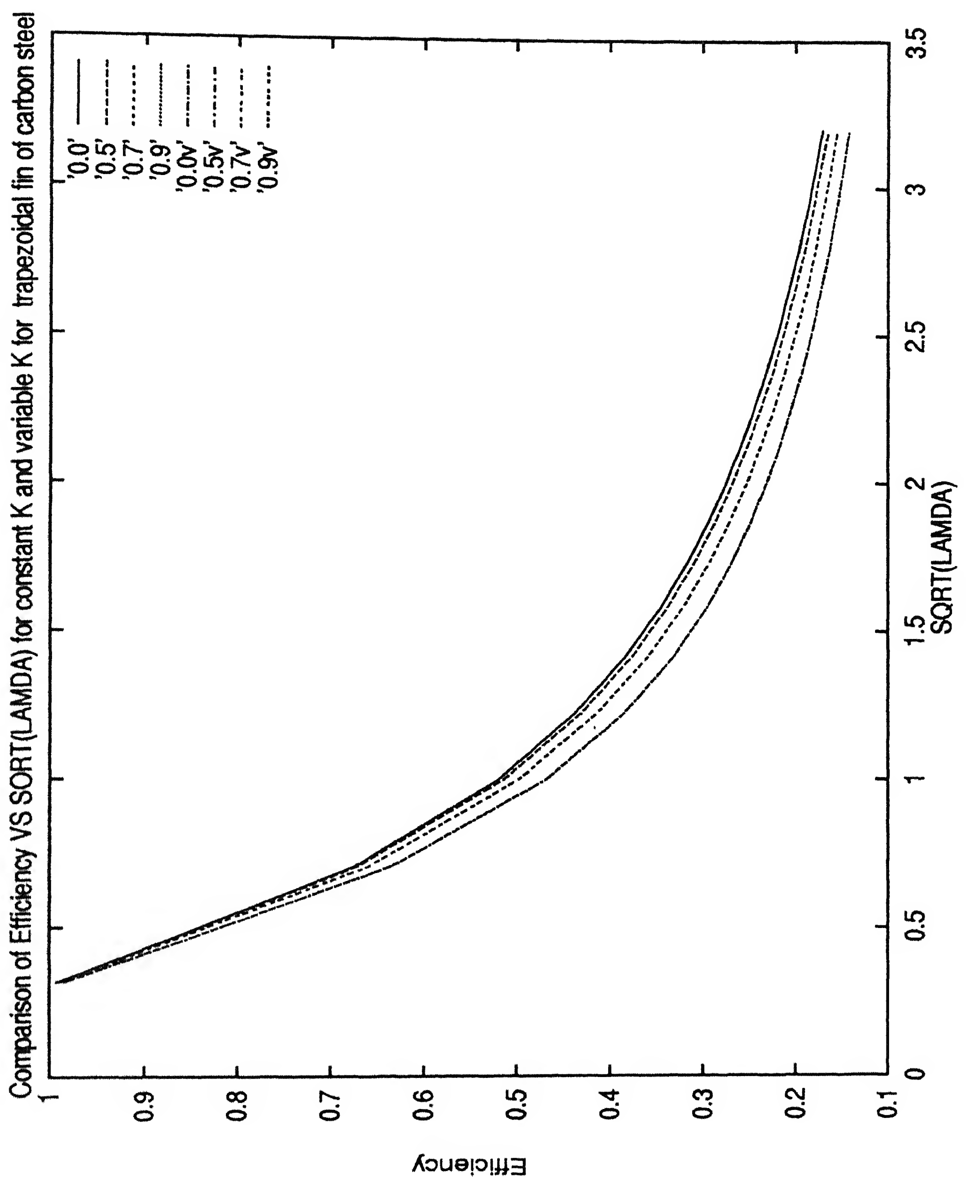

Figure 3.11: Comparison of Efficiency Vs$\sqrt{\lambda}$ for constant conductivity and variable conductivity of trapezoidal of copper

'0.0' indicates $\frac{t_s}{t_0} = 0.0$, '0.5 'indicates $\frac{t_s}{t_0} = 0.5$, '0.7' indicates $\frac{t_s}{t_0} = 0.7$

'0.9 'indicates $\frac{t_s}{t_0} = 0.9$,

For variable conductivity material '0.0v' indicates $\frac{t_s}{t_0} = 0.0$,

'0.5v' indicates $\frac{t_s}{t_0} = 0.5$, '0.7v' indicates $\frac{t_s}{t_0} = 0.7$, '0.9v' indicates $\frac{t_s}{t_0} = 0.9$

$$L = 0.1m$$

EFFICIENCY VS $\sqrt{\lambda}$

Using

$$a = 0.0001875$$

$$b = -0.001125$$

$$c = 0.000005$$

$$L = 0.1m$$

For carbon steel

Base temperature is taken as 323K.

$$\lambda = \frac{\epsilon \sigma t_0^3 L^2}{2 k_1 a} \tag{3.5}$$

k_1 is the thermal conductivity at base temperature.

See Fig. 3.13 .

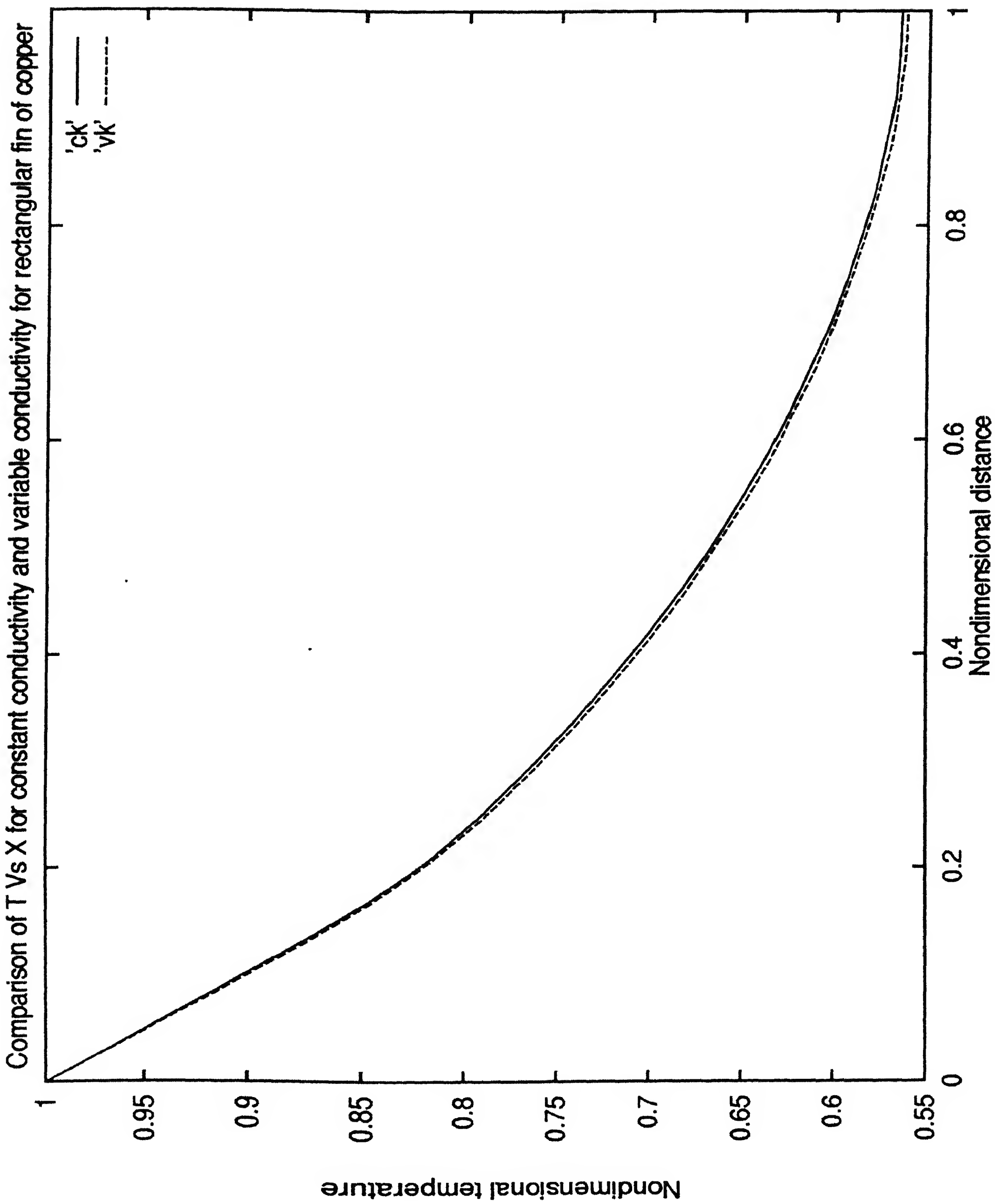

Figure 3.12: Comparison of T Vs X for constant and variable thermal conductivity for copper for parabolic profile
'ck' indicates constant conductivity.'vk' indicates variable

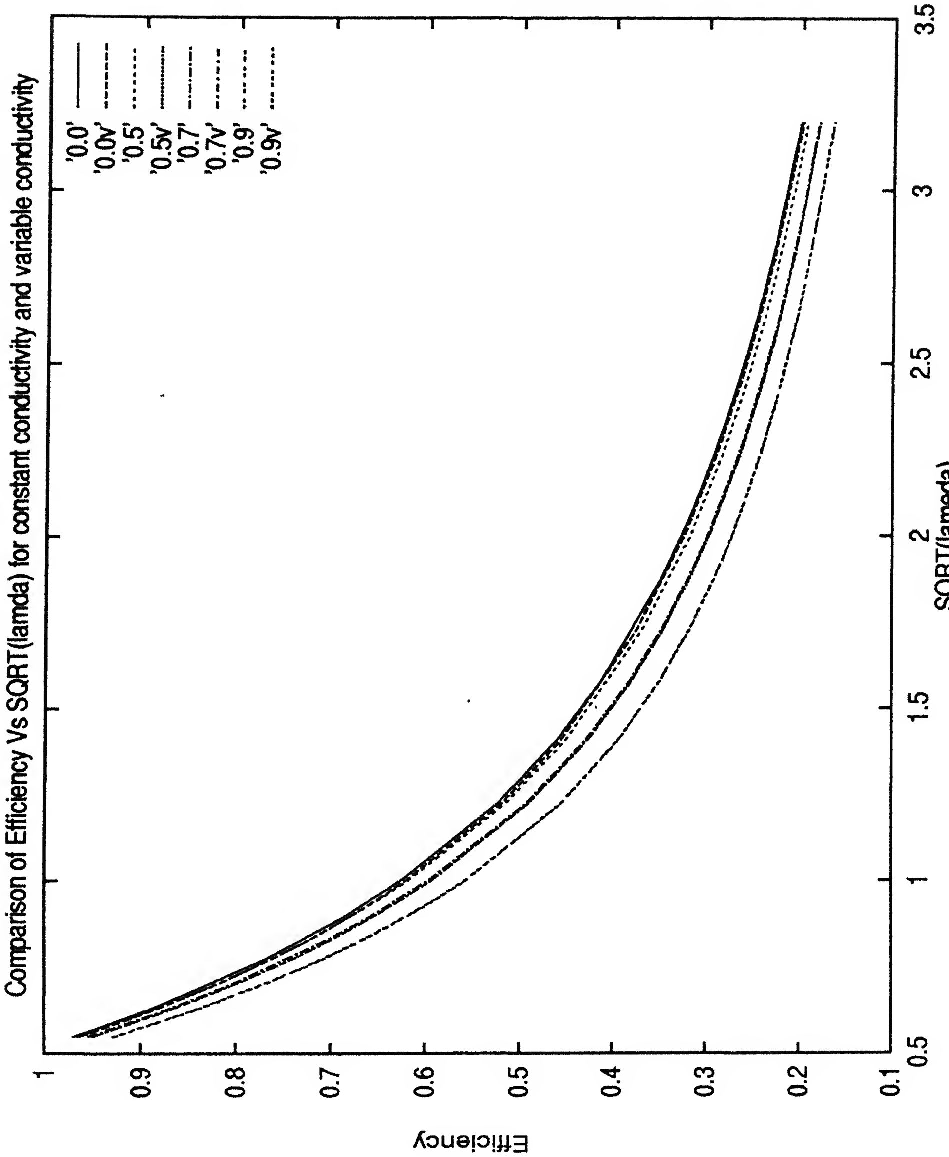

Figure 3.13: Comparison of Efficiency Vs $\sqrt{\lambda}$ for constant conductivity and variable conductivity for parabolic fin
'0.0' indicates $\frac{t_s}{t_0} = 0.0$, '0.5 'indicates $\frac{t_s}{t_0} = 0.5$, '0.7' indicates $\frac{t_s}{t_0} = 0.7$
'0.9 'indicates $\frac{t_s}{t_0} = 0.9$,
For variable conductivity material '0.0v' indicates $\frac{t_s}{t_0} = 0.0$,
'0.5v' indicates $\frac{t_s}{t_0} = 0.5$, '0.7v' indicates $\frac{t_s}{t_0} = 0.7$, '0.9v' indicates $\frac{t_s}{t_0} = 0.9$.

Chapter 4

OPTIMIZATION OF HEAT TRANSFER FROM THE FIN

4.1 BASIC OBJECTIVES OF OPTIMIZATION

Radiating fins are used in spacecraft and the space vehicles for rejecting on-board waste heat to deep space . Since weight is at premium , it is important to have minimum fin mass in these application or maximization of heat transfer per unit weight of the fin.

4.1.1 DEFINITION OF OBJECTIVE FUNCTION

The objective function is the heat transfer from the fin per unit mass.

$$q1 = -\frac{k_1 W t_0}{\rho V L}(\frac{dT}{dX})_{i=1} \tag{4.1}$$

V is the volume of the fin.

For rectangular fin

$$V = WL \tag{4.2}$$

For trapezoidal fin

$$V = (W_1 + W_2)\frac{L}{2} \tag{4.3}$$

For parabolic fin

$$V = aL + b\frac{L^2}{2} + c\frac{L^3}{3} \tag{4.4}$$

For all cases , the base temperature (t_0) is taken as 323K.

4.1.2 DESIGN VARIABLES

The design variables are the dimension of the fin for rectangular fin W and L ,for trapezoidal fin W_1, $W_2$and L . The design variables for parabolic profile are a , b , c and L . The heat transfer per unit mass is also function of the thermal conductivity .

4.1.3 LIMITATION AND CONSTRAINTS

The constraint is the volume of the fin . The limitation is the melting point of the fin material.

4.2 OPTIMIZATION METHOD

EXHAUSTIVE SEARCH:-The method consists of calculating the value of objective function at the values of variable that are spaced uniformly through out the domain of interest.The optimum is obtained by eliminating regions not containing the maximum.

The interval of uncertainty for 1^{st} 2^{nd} ......n^{th} variables are I_1, I_2 .....I_N.

$$I_1 = \frac{I_{O1}}{N_1 + 1} \tag{4.5}$$

$$I_2 = \frac{I_{O2}}{N_2 + 1} \tag{4.6}$$

$$I_N = \frac{I_{ON}}{N_N + 1} \tag{4.7}$$

Where I_{O1} , I_{O2} ——I_{ON} are the range of interest for 1^{st} , 2^{nd} ——N^{th} variables.

Where number of observation are $N_1 \times N_2 \times ------ \times N_N$.

4.3 RESULTS FOR ALL THREE TYPE OF FINS FOR DIFFERENT MATERIALS

4.3.1 FOR RECTANGULAR FIN OPTIMUM DIMENSIONS FOR DIFFERENT MATERIAL

The calculated optimum dimensions of fin

RANGE OF INTEREST

Range of L (0.075m,0.30m)

Range of W (0.0001m ,0.0005m)

Volume of the fin $=2.8125 \times 10^{-5} m^3$

Type of material	Width(m)	Length(m)	Heat transfer(W/kg)
Aluminium	0.000163	0.172546	396.242
Carbon steel	0.0002845	0.0988567	78.51465
Copper	0.000133	0.2114662	147.0347

Table 4.1: Optimum dimension for Rectangular fins

4.3.2 FOR TRAPEZOIDAL FIN

The calculated optimum dimensions of fin

RANGE OF INTEREST

Range of L (0.075m,0.30m))

Range of W_1 (0.0001m, 0.0005m)

Range of W_2 (0.0001m,0.0005m)

Volume of the fin $=2.8125 \times 10^{-5} m^3$

Type of material	W_1	W_2	Length	Heat transfer(W/kg)
Aluminium	0.000267	0.0001	0.153270	556.62384
Carbon steel	0.000468	0.0001	0.0990316	124.3621
Copper	0.000219	0.0001	0.176333	193.5711

Table 4.2: Optimum dimension for Trapezoidal fins

4.3.3 FOR PARABOLIC FIN

The calculated optimum dimensions of fin

RANGE OF INTEREST

Range of L (0.075m,0.30m)

Range of a (0.0001 ,0.0005)

Range of b (-0.001 , 0.0)

Range of c (-0.02 ,0.02)

Volume of the fin $=2.8125 \times 10^{-5} m^3$

Type of material	a	b	c	L (m)	Fin width at tip (m)	$q1$(W/kg)
Aluminium	0.000121	-0.001	0.0029	0.25	0.000104	725.47
Carbon steel	0.0002207	-0.001	-0.00837	0.095	0.0001004	113.226
Copper	0.000184	-0.001	-0.00212	0.229	0.001	250.94

Table 4.3: Optimum dimension for parabolic fins

4.4 CHOICE OF MATERIAL FOR EACH TYPE OF FINS

Aluminium is the most suitable material for each type of fin because it gives maximum heat transfer per unit mass . Which is expected also due to its

high conductivity and least density.

Chapter 5

CONCLUSION

A heat transfer analysis of radiating fins of various materials and various geometries is presented. Both constant and variable conductivity fin materials have been studied. The results for rectangular fin show excellent agreement with an earlier NASA work. The second part of this thesis deals with finding dimensions corresponding to the maximum heat transfer per unit weight of the fin material having the temperature dependent conductivity. An exhaustive search method has been used. It is found that aluminium is the most suitable material. Also, trapezoidal and parabolic fins turned out to better than rectangular fins for efficient heat transfer.

Chapter 6

REFERENCES

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2. Chung, B. T. F., and Zhang, B. X., 1991a, " Minimum Mass Longitudinal Fins with Radiation Interaction at the Base," *JOURNAL OF ERANKLIN INSTITUTE*, Vol. 328, pp. 143-161.

3. Chung, B. T. F., and Zhang, B. X., 1991b, "Optimization of Radiating Fins Array Including Mutual Irradiation Between Radiator Elements," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 113, pp. 814-822.

4. Chung, B. T. F., Zhang, B. X., and Lee, E. T., 1996, "A Multi-Objective Optimization of Radiative Fin Array Systems in a Fuzzy Environment," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 118, pp. 642-659.

5. Karlekar, B. V., and Chao, B. T., 1963, "Mass Minimization of Ra-

Appendix A

Five point forward differencing

Five point forward differential for calculating temperature gradient for calculation of heat transfer.

$$(\frac{dT}{dX})_i = \frac{-25T_i + 48T_{i+1} - 36T_{i+2} + 16T_{i+3} - 3T_{i+4}}{12\Delta X}$$

Appendix B

Properties of material

Thermal conductivity k for aluminum $= (207.0615 - 0.0054894t + 2.9267 \times 10^{-4}t^2)$ W/m.K

Thermal conductivity k for carbon steel $= (40.882 - 0.0141t\ )$ W/m.K

Thermal conductivity k for copper $=(390.9975 - 0.0511051t\ )$ W/m.K

t is in C.

The above thermal conductivity values are applicable in the range of $0 - 400$ C

Density of aluminium $= 2707\ \frac{m^3}{kg}$

Density of carbon steel $= 7833\frac{m^3}{kg}$

Density of copper $= 8954\frac{m^3}{kg}$

$\epsilon = 0.50$

$\alpha = 0.10$

Irradiation G =315.4 $\frac{W}{m^2}$

Appendix C

Computer Programs

C.1 For Rectangular Fin

```
c THE PROGRAM FOR RECTANGULAR FIN
  c THE OUTPUT CAN BE SEEN BY VIEWING 'OUTP'FILE.
  c THIS PROGRAM MUST BE COMPILED ALONGWITH tdma.f.
  c at REPRESENT THE NON DIMENSIONAL TEMPERATURE
  c att REPRESENT THE NON DIMENSIONAL TEMPERATURE IN
PREVIOUS ITERATION
        DIMENSION a(121),b(121),c(121),d(121),at(121),ak(121),att(121)
        OPEN(UNIT=76,FILE='OUTP')
        read *, n
  c*******PROPERTIES AND DIMENSION OF THE FIN ********************
        e = 0.5
```

```
alpha = 0.1
t0 = 323.14
al = 0.075
a1 = 40.882
b1 = 0.0141
w = 0.000375
g = 315.4
sigma = 5.67e-8
ts = (alpha*g/e*sigma)**0.25
const =(w/(e*sigma*(al**2)*(t0**3)))
do 10 i= 1,n
c*******CALCULATION OF THERMAL CONDUCTIVITY AT BASE
TEMPERATURE FOR EACH GRID
ak(i) = a1-(b1*(t0-273.14))
c*******ASSUMING THE PREVIOUS ITERATION NON DIMENSIONAL
TEMP. FOR EACH GRID
att(i) = 1.0
10 continue
45 ats = ts/t0
dx = 1.0/float(n-1)
c*******MAIN DIAGONAL COFFICIENT OF TDMA ************************
b(1)= 1.
do 30 i= 2,n
```

```
      b(i) = ((-2*const*ak(i)/dx**2)-att(i)**3)
30    continue
c*******SUPER-DIAGONAL COFFICIENT OF TDMA **********************
      c(1) = 0
      do 40 i= 2,n-1
      c(i) =(const*ak(i)/dx**2) + const*(ak(i+1) - ak(i-1))/4.*dx**2
40    continue
c*******SUB-DIOGONAL COFFICIENT OF TDMA *************************
      a(2) = 0.0
      do 50 i= 3,n-1
      a(i) = ((const*ak(i)/dx**2) - (const*(ak(i+1)-ak(i-1))/2.*dx))
50    continue
      a(n) = 2*const*ak(n)/dx**2
c*******COFFICIENT OF RIGHT HAND COLUMN VECTOR
      d(1) = 1.
      d(2) = -1.0*((const*ak(2)/dx**2)-(const*(ak(3)-ak(2))/4.*dx**2))
- ats**4
      do 60 i = 3,n
      d(i) = -ats**4
60    continue

      CALL TDMA(1,n,A,B,C,D,AT)
      do  i=1,n
```

```
c*******CALCULATION OF THERMAL CONDUCTIVITY AT NEW
TEMP. FOR EACH GRID POINT
      ak(i) = a1-(b1*(t0*at(i)-273.14))
      err=abs(at(i)-att(i))
      att(i)=at(i)
      if (err.le.0.001) go to 55
      go to 45
55    enddo
c****** CALCULATION OF TEMPERATURE GRADIENT, HEAT TRANS-
FER EFFICIENCY **********
      q = (ak(1)*w*t0*(at(3)-4.*at(2)+ 3.*at(1))/(2.*dx*al))
      effeciency = q/((e*sigma*(t0)**4-alpha*g)*al)
      WRITE(76,9)
9     FORMAT(4x,'THE SOLUTION BY TDMA'/)
      WRITE(76,2)(at(i),i=1,n)
2     FORMAT(2x,5F6.3)
      write(76,8)
8     format(4x,'HEAT LOSS BY THE FIN'/)
      write(76,3) q
3     format(2x,f9.3)
      write(76,7)
7     format(4x,'effeciency of fin')
      write(76,1) effeciency
```

```
1     format(2x,f6.3)
      STOP
      END


c THE NAME OF THE PROGRAM:tdma.f
c SUBROUTINE FOR SOLVING A SYSTEM OF LINEAR
c SIMULTANEOUS ALGEBRAIC EQUATIONS HAVING
c A TRIDIAGONA COEFFICIENT MATRIX.
c THE EQUATIONS ARE NUMBERED FROM I THROUGH N,
c AND THEIR SUB-DIAGONAL,DIAGONAL AND SUPER-DIAGONAL
c ELEMENTS ARE STORED IN THE ARRAYS A,B,AND C.THE
c RIGHT-HAND COLUMN VECTOR ELEMENTS ARE STORED
c IN THE ARRAY D.THE COMPUTED SOLUTION VECTOR X(I)....X(N)
c IS STORED IN THE ARRAY X.

      SUBROUTINE TDMA(I,N,A,B,C,D,X)
      DIMENSION A(1),B(1),C(1),D(1),X(1),BETA(101),GAMMA(101)
c ****** COMPUTE INTERMEDIATE ARRAYS BETA AND GAMMA
******
      BETA(I)=B(I)
      GAMMA(I)=D(I)/BETA(I)
      I1=I+1
```

```
      DO 1 J=I1,N
      BETA(J)=B(J)-A(J)*C(J-1)/BETA(J-1)
    1 GAMMA(J)=(D(J)-A(J)*GAMMA(J-1))/BETA(J)
c ****** COMPUTE THE SOLUTION VECTOR X ******
      X(N)=GAMMA(N)
      N1=N-I
      DO 2 K=1,N1
      J=N-K
    2 X(J)=GAMMA(J)-C(J)*X(J+1)/BETA(J)
      END
```

C.2 For Trapezoidal Fin

```
c THE PROGRAM FOR TRAPEZOIDAL FIN
  c THE OUTPUT CAN BE SEEN BY VIEWING 'OUTP'FILE.
  c THIS PROGRAM MUST BE COMPILED ALONGWITH tdma.f.
  c at(i) REPRESENT THE NON DIMENSIONAL TEMPERATURE
  c ak(i) REPRESENT THE THERMAL CONDUCTIVITY AT ANY
GRID POINT
  c att(i) REPRESENT THE NON DIMENSIONAL TEMP. IN PREVI-
OUS ITERATION
      DIMENSION a(121),b(121),c(121),d(121),at(121),ak(121),att(121),x(50)
      DIMENSION c1(100),c2(100),d1(111),d2(100),a1(100),a2(100),s(100)
```

```
OPEN(UNIT=76,FILE='OUTP')
read *,n
c ****** PROPERTIES AND DIMENSION OF THE FIN*************************
e = 0.500
w1 = 0.000375
alpha = 0.10
v = 2.8125e-05
u1 = 390.9975
u2 = 0.0511051
t0 = 323.
dens = 8954
w2 =0.000100
al = (2.*v/(w1+w2))
g = 315.4
sigma = 5.67e-8
ts = (alpha*g/e*sigma)**0.25
const =(1./(e*sigma*(al**2)*(t0**3)))
dx = 1./float(n-1)
do 10 i= 1,n
s(1) = w1
s(i+1) = s(i)-(w1-w2)*dx
c******CALCULATION OF THERMAL CONDUCTIVITY AT BASE
TEMPERATURE FOR EACH GRID
```

```
        ak(i) = u1-(u2*(t0-273.14))
c*******ASSUMING THE PREVIOUS ITERATION NON DIMENSIONAL
TEMPERATURE FOR EACH GRID
        att(i) = 1.0
10      continue
45      ats = ts/t0
c*******MAIN DIAGONAL COFFICIENT OF TDMA *************************
        b(1)= 1.
        do 30 i= 2,n
        b(i) = ((-2*s(i)*const*ak(i)/dx**2)-att(i)**3)
30      continue
c******SUPER-DIAGONAL COFFICIENT OF TDMA *************************
        c(1) = 0.0
           c1(i) =(const*ak(i)*s(i)/dx**2) + s(i)*const*(ak(i+1) - ak(i-
1))/4.*dx**2
        c2(i) = const*ak(i)*(w1-w2)/2*dx
        c(i) = c1(i)-c2(i)
40      continue
c*******SUB-DIOGONAL COFFICIENT OF TDMA****************************
        a(2) = 0.0
        do 50 i= 3,n-1
       a1(i) = s(i)*((const*ak(i)/dx**2) - (const*(ak(i+1)-ak(i-1))/2.*dx))
        a2(i) = const*ak(i)*(w1-w2)/2.*dx
```

```
      a(i) = a1(i)+a2(i)
50    continue
      a(n) = 2*const*ak(n)*s(n)/dx**2
c*******COFFICIENT OF RIGHT HAND COLUMN VECTOR*****************
      d(1) = 1.
    d1(2) = -1.0*((const*ak(2)/dx**2)-(const*(ak(3)-ak(2))/4.*dx**2))*s(2)
      d2(2) = -(const*ak(2)*(w1-w2)/2.*dx)-ats**4
      d(2) = d1(2)+d2(2)
      do 60 i = 3,n
      d(i) = -ats**4
60    continue

      CALL TDMA(1,n,A,B,C,D,AT)
      do  i=1,n
c****** CALCULATION OF THERMAL CONDUCTIVITY AT NEW
TEMPERATURE FOR EACH GRID POINT
      ak(i) = u1-(u2*(t0*at(i)-273.14))
      err=abs(at(i)-att(i))
      att(i)=at(i)
      if (err.le.0.0001) go to 55
      go to 45
55    enddo
c*******CALCULATION OF TEMPERATURE GRADIENT, HEAT TRANS-
```

FER AND EFFICIENCY*********

```
      anu=-(-25.*at(1)+48.*at(2)-36.*at(3)+16.*at(4)-3*at(5))
      den = 12.*dx*al
      div = anu/den
      q1 = ak(1)*w1*t0*div
      x(1) = 0.0
      do i=1,n-1
      x(i+1)=x(i)+dx
      enddo
      effeciency = q1/((e*sigma*(t0)**4-alpha*g)*al)
      q = q1/(dens*v)
      print *, w1,w2,al,q,effeciency
      WRITE(76,9)
 9    FORMAT(4x,'THE SOLUTION BY TDMA'/)
      WRITE(76,2)(x(i),at(i),i=1,n)
 2    FORMAT(2x,2F6.3)
      write(76,8)
 8    format(4x,'HEAT LOSS BY THE FIN'/)
      write(76,3) q
 3    format(2x,f15.3)
      write(76,7)
 7    format(4x,'effeciency of fin')
      write(76,1) effeciency
```

```
1     format(2x,f9.3)
      STOP
      END

c THE NAME OF THE PROGRAM:tdma.f
c SUBROUTINE FOR SOLVING A SYSTEM OF LINEAR
c SIMULTANEOUS ALGEBRAIC EQUATIONS HAVING
c A TRIDIAGONA COEFFICIENT MATRIX.
c THE EQUATIONS ARE NUMBERED FROM I THROUGH N,
c AND THEIR SUB-DIAGONAL,DIAGONAL AND SUPER-DIAGONAL
c ELEMENTS ARE STORED IN THE ARRAYS A,B,AND C.THE
c RIGHT-HAND COLUMN VECTOR ELEMENTS ARE STORED
c IN THE ARRAY D.THE COMPUTED SOLUTION VECTOR X(I)....X(N)
c IS STORED IN THE ARRAY X.
      SUBROUTINE TDMA(I,N,A,B,C,D,X)
      DIMENSION A(1),B(1),C(1),D(1),X(1),BETA(101),GAMMA(101)
c ****** COMPUTE INTERMEDIATE ARRAYS BETA AND GAMMA
******
      BETA(I)=B(I)
      GAMMA(I)=D(I)/BETA(I)
      I1=I+1
      DO 1 J=I1,N
      BETA(J)=B(J)-A(J)*C(J-1)/BETA(J-1)
```

```
 1   GAMMA(J)=(D(J)-A(J)*GAMMA(J-1))/BETA(J)
c ****** COMPUTE THE SOLUTION VECTOR X ******
     X(N)=GAMMA(N)
     N1=N-I
     DO 2 K=1,N1
     J=N-K
 2   X(J)=GAMMA(J)-C(J)*X(J+1)/BETA(J)
     END
```

C.3 For Parabolic Fin

```
c THE PROGRAM FOR PARABOLIC FIN
  c THE OUTPUT CAN BE SEEN BY VIEWING 'OUTP'FILE.
  c THIS PROGRAM MUST BE COMPILED ALONGWITH tdma.f.
  c at REPRESENT THE NON DIMENSIONAL TEMPERATURE
  c att REPRESENT THE NON DIMENSIONAL TEMPERATURE IN
PREVIOUS ITERATION
      DIMENSION a(121),b(121),c(121),d(121),at(121),ak(121),att(121)
     DIMENSION c1(100),c2(100),d1(111),d2(100),a1(100),a2(100),s(100)
      DIMENSION x(121),z(121),xx(50)
      OPEN(UNIT=76,FILE='OUT')
      read *, v0
  c*******THE PROPERTIES AND DIMENSION OF THE FIN *********************
```

```
n = 25
e = 0.500
alpha = 0.10
vol = 2.8125e-05
al = 0.229
v1 = -0.0001000
v2 = (1./(al*al*al))*((vol/2.)-(v0*al)-(v1*al*al/2.))
t0 = 823.
u1 = 390.9975
u2 = 0.0511051
dens = 8954
w1 = 2.*v0
w2 = 2.0*(v0+v1*al+v2*al*al)
g = 315.4
sigma = 5.67e-8
ts = (alpha*g/e*sigma)**0.25
const =(1./(e*sigma*(al**2)*(t0**3)))
dx = 1./float(n-1)
do 10 i= 1,n
x(1) = 0.0
x(i+1)=x(i)+dx
s(i) = v0 + (v1*x(i)*al)+(v2*((x(i)*al)**2))
z(i) = v1*al+2.*v2*x(i)*al*al
```

```
c       ak(i) = u1
c*******CALCULATION OF THERMAL CONDUCTIVITY AT BASE
TEMPERATURE FOR EACH GRID POINT
        ak(i) =  u1-(u2*(t0-273.14))
c******ASSUMING THE PREVIOUS ITERATION NON DIMENSIONAL
TEMPERATURE FOR EACH GRID
        att(i) = 1.0
10      continue
45      ats = ts/t0
c*******MAIN DIAGONAL COFFICIENT OF TDMA *************************
        b(1)= 1.
        do 30 i= 2,n
        b(i) = ((-4.*s(i)*const*ak(i)/dx**2)-att(i)**3)
30      continue
c****** SUPER-DIAGONAL COFFICIENT OF TDMA*************************
        c(1) = 0
        do 40 i= 2,n-1
        c1(i) =(ak(i)*s(i)/dx**2) + s(i)*(ak(i+1) - ak(i-1))/4.*dx**2
        c2(i) = ak(i)*z(i)/2*dx
        c(i) = (c1(i)+c2(i))*2.*const
40      continue
c ******SUBDIOGONAL COFFICIENT OF TDMA ****************************
        a(2) = 0.0
```

```
      do 50 i= 3,n-1
     a1(i) = s(i)*((const*ak(i)/dx**2)-(const*(ak(i+1)-ak(i-1))/4.*dx*dx))
      a2(i) = const*ak(i)*z(i)/2.*dx
      a(i) = (a1(i)-a2(i))*2.
50    continue
      a(n) = 4*const*ak(n)*s(n)/dx**2
c*******COFFICIENT OF RIGHT HAND COLUMN VECTOR *****************
      d(1) = 1.
     d1(2) = -1.0*((const*ak(2)/dx**2)-(const*(ak(3)-ak(2))/4.*dx**2))*s(2)
      d2(2) = 2.*(const*ak(2)*z(i)/2.*dx)-ats**4
      d(2) = 2.*d1(2)+d2(2)
      do 60 i = 3,n
      d(i) = -ats**4
60    continue

      CALL TDMA(1,n,A,B,C,D,AT)
      do  i=1,n
c*******CALCULATION OF THERMAL CONDUCTIVITY AT NEW
TEMP. FOR EACH GRID POINT
      ak(i) = u1 - u2*(at(i)*t0-273.14)
      err=abs(at(i)-att(i))
      att(i)=at(i)
      if (err.le.0.0001) go to 55
```

```
      go to 45
55    enddo
c*******CALCULATION OF TEMPERATURE GRADIENT, HEAT TRANS-
FER AND EFFICIENCY
      anu=-(-25.*at(1)+48.*at(2)-36.*at(3)+16.*at(4)-3*at(5))
      den = 12.*dx*al
      div = anu/den
      q1 = ak(1)*w1*t0*div
      eff = q1/((e*sigma*(t0)**4-alpha*g)*al)
      q = q1/(dens*vol)
      xx(1) = 0.0
      do i = 1, n-1
      xx(i+1) = xx(i) + dx
      enddo
      print 25,v0 ,v1,v2,al,w2,q,eff
25    format(/7(f11.6))
      WRITE(76,9)
 9    FORMAT(4x,'THE SOLUTION BY TDMA'/)
      WRITE(76,2)(xx(i), at(i),i=1,n)
 2    FORMAT(2x,2F6.3)
      write(76,8)
 8    format(4x,'HEAT LOSS BY THE FIN'/)
      write(76,3) q
```

```
3     format(2x,f15.3)
      write(76,7)
7     format(4x,'effeciency of fin')
      write(76,1) effeciency
1     format(2x,f9.3)
      STOP
      END

c THE NAME OF THE PROGRAM:tdma.f
c SUBROUTINE FOR SOLVING A SYSTEM OF LINEAR
c SIMULTANEOUS ALGEBRAIC EQUATIONS HAVING
c A TRIDIAGONA COEFFICIENT MATRIX.
c THE EQUATIONS ARE NUMBERED FROM I THROUGH N,
c AND THEIR SUB-DIAGONAL,DIAGONAL AND SUPER-DIAGONAL
c ELEMENTS ARE STORED IN THE ARRAYS A,B,AND C.THE
c RIGHT-HAND COLUMN VECTOR ELEMENTS ARE STORED
c IN THE ARRAY D.THE COMPUTED SOLUTION VECTOR X(I)....X(N)
c IS STORED IN THE ARRAY X.
      SUBROUTINE TDMA(I,N,A,B,C,D,X)
      DIMENSION A(1),B(1),C(1),D(1),X(1),BETA(101),GAMMA(101)
c ****** COMPUTE INTERMEDIATE ARRAYS BETA AND GAMMA
******
      BETA(I)=B(I)
```

```
      GAMMA(I)=D(I)/BETA(I)
      I1=I+1
      DO 1 J=I1,N
      BETA(J)=B(J)-A(J)*C(J-1)/BETA(J-1)
 1    GAMMA(J)=(D(J)-A(J)*GAMMA(J-1))/BETA(J)
c ****** COMPUTE THE SOLUTION VECTOR X ******
      X(N)=GAMMA(N)
      N1=N-I
      DO 2 K=1,N1
      J=N-K
 2    X(J)=GAMMA(J)-C(J)*X(J+1)/BETA(J)
      END
```